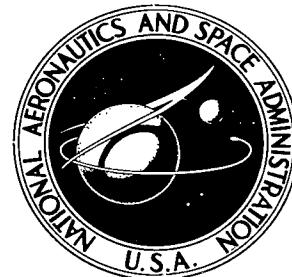


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SYSTEMS IDENTIFICATION USING
A MODIFIED NEWTON-RAPHSON METHOD -
A FORTRAN PROGRAM

by

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SYSTEMS IDENTIFICATION USING
A MODIFIED NEWTON-RAPHSON METHOD – A FORTRAN PROGRAM

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SUMMARY

A digital computer program written in FORTRAN is offered which computes a maximum-likelihood estimate of the parameters of a linear, state space model. For the case considered, the maximum-likelihood estimate can be identical to one which minimizes simultaneously the weighted mean-square difference between the computed and measured response of a system, and the weighted square of the difference between the estimated and a priori parameter values. A modified Newton-Raphson or quasilinearization method is used to perform the minimization which typically requires several iterations. The modification of the Newton-Raphson method was made in the interest of reduced computation and program simplicity. A starting technique is used which insures convergence for any initial values of the unknown parameters. A Cramèr-Rao bound is used to indicate the variance of the estimated parameter values. Although the primary application of the program has been to determine aircraft stability derivatives from flight data, it is directly applicable to identification of any system which can be described by a linear, constant-coefficient model. The intent of this paper is to describe the program and its operation in sufficient detail to enable the user to apply the program to his particular problem with a minimum of difficulty.

INTRODUCTION

Problems in systems identification, in which models are constructed from response data, continue to receive considerable attention because of their importance and because of the difficulties involved. A few years ago linear regression methods were employed almost exclusively for modeling systems. The most often used methods were least squares (ref. 1) and Shinbrot's method (ref. 2). Unfortunately, these methods often required measurements of variables not generally available and, at best, would match the

differential equations of motion to the response data. Too often the response calculated from the resulting models did not match the measured response well. It was recognized that it would be better to match the model response to the measured response per se, but that problem was difficult to solve because it was nonlinear. Only after experience had been gained in solving large numbers of nonlinear, simultaneous equations were these systems identification problems solved by using a modified Newton-Raphson method (ref. 3) (or quasilinearization (ref. 4)). A second order was used because of its superior convergence properties. Several algorithms (refs. 5 to 9) have now been developed and numerous applications have been made (refs. 10 and 11, for example), particularly to the problem of determining stability derivatives of airplanes.

The modified Newton-Raphson (quasilinearization) method is applicable for modeling any linear system with constant coefficients. The algorithm includes features which enable the use of a priori information such as wind-tunnel measurements. The initial conditions and bias terms can also be treated as unknown parameters.

The purpose of this paper is to make available in a form for general use the program that has been successfully used for determination of aerodynamic derivatives in references 10 and 11. It is intended that the accompanying description, program, and example test case would enable a prospective user to run the test case first and then substitute the data for his particular application with relative ease.

SYMBOLS

Program variables are given in a separate section.

- A systems matrix (MX by MX)
- B control matrix (MX by MU)
- c parameter vector (MC by 1)
- c_0 a priori parameter vector (MC by 1)
- \hat{c} estimate of parameter vector (MC by 1)
- D_1 weighting matrix for measured response
- D_2 weighting matrix for a priori estimate vector
- $E\{\cdot\}$ expected value

F	state transformation matrix (MZ by MX)
G	control transformation matrix (MZ by MU)
h	time increment
I	identity matrix
i,k	time and iteration indices, respectively
J	cost function to be minimized
N	number of time points
P()	probability
p	roll rate
r	yaw rate
T	record length, sec; as superscript, transpose
t	time, sec
u	control vector
x	state vector
\dot{x}	time derivative of state vector
y	calculated response vector
z	measured response vector
β	sideslip angle
δ_a	aileron deflection
δ_r	rudder deflection

ϕ	bank angle
$\nabla_c(\cdot)$	first variation (gradient) with respect to parameter vector
$\nabla_c^2(\cdot)$	second variation (gradient) with respect to parameter vector

Dots over symbols denote derivatives with respect to time.

The program variables used in the definitions of the symbols are as follows:

MC	number of unknown parameters
MU	number of control variables
MX	number of state variables
MZ	number of response variables

SYSTEMS IDENTIFICATION

The systems identification problem of determining the parameters of a linear, constant-coefficient model will be considered for several types of estimates. It will be shown that maximum-likelihood estimates can be identical to those which minimize the mean-square response error. The subject algorithm, therefore, can be used to obtain a variety of similar estimates. Attention is also given to the calculation of the gradient that is involved in the algorithm and to the Cramèr-Rao bound which indicates the variance of the estimates.

Problem Statement

The problem considered is that of determining the values of certain model parameters which are best with regard to a particular criterion, if the input and noise measurements of the response of a linear, constant-coefficient system are given. The system to be identified is defined by the following equations:

$$\dot{x} = Ax + Bu \quad (1)$$

$$y = Fx + Gu + b \quad (2)$$

$$z = y + n \quad (3)$$

where

- x state vector
- u control vector
- y calculated response variable
- b constant-bias vector
- n noise vector
- z measured response variable

The unknown parameters will form a vector c . The matrices A , B , F , and G and the vectors b and $x(0)$ are functions of c .

Minimum-Response Error Estimate

One criterion that is often used in systems identification is the mean-square difference between the measured response and that given by the model. A cost function which is proportional to the mean-square error can be written as

$$J = \sum_{i=1}^N (z_i - y_i)^T D_1 (z_i - y_i) \quad (4)$$

where D_1 is a weighting matrix and i is a time index. The summation approximates a time integral. The estimate of c is then

$$\hat{c} = \underset{c}{\text{ARG MIN}}(J) \quad (5)$$

where ARG MIN means that vector c which minimizes the cost function J .

Linearize the calculated response y with respect to the unknown parameter vector c :

$$y_i = y_{i_0} + \nabla_c y_i (c - c_0) \quad (6)$$

where

y_{i_0} nominal response calculated by using c_0

$\nabla_c y_i$ gradient of y with respect to c

c_0 nominal c vector

Substituting y_i into the expression for J and solving for the value of c which minimizes J yields

$$\hat{c} = c_0 + \left[\sum_{i=1}^N \nabla_c y_i^T D_1 \nabla_c y_i \right]^{-1} \left[\sum_{i=1}^N \nabla_c y_i^T D_1 (z_i - y_{i_0}) \right] \quad (7)$$

If this relationship is applied iteratively to update the calculated nominal response and its gradient with respect to the unknown parameter vector, the minimum-response error estimate \hat{c} will result. The method has been called quasilinearization (refs. 6 and 7) and modified Newton-Raphson (refs. 5 and 10). The latter seems more appropriate since the Newton-Raphson (ref. 3) method would give

$$c_{k+1} = c_k + \left[\nabla_c^2 J_k \right]^{-1} \left[\nabla_c J_k \right] \quad (8)$$

where

$$\nabla_c J_k = -2 \sum_{i=1}^N \nabla_c y_{i_k}^T D_1 (z_i - y_{i_k}) \quad (9)$$

$$\nabla_c^2 J_k = 2 \sum_{i=1}^N \nabla_c y_{i_k}^T D_1 \nabla_c y_{i_k} + 2 \sum_{i=1}^N \nabla_c^2 y_{i_k}^T D_1 (z_i - y_{i_k}) \quad (10)$$

The second term of $\nabla_c^2 J_k$ diminishes as the response error $(z_i - y_{i_k})$ diminishes. The modified Newton-Raphson method is identical to quasilinearization if the term is neglected.

Maximum Conditional Likelihood Estimate

Another criterion that is often used is to select c to maximize the likelihood of the measured response when c is given:

$$\hat{c} = \operatorname{ARG MAX}_c \{P(z|c)\} \quad (11)$$

If the noise n has a Gaussian distribution, the probability distribution of the measured response at any single time point can be written as

$$P(z_i|c) = \frac{1}{(2\pi)^{(MZ)/2} |M_1|^{1/2}} \exp \left[-\frac{1}{2} (z_i - y_i)^T M_1^{-1} (z_i - y_i) \right] \quad (12)$$

where

$$M_1 = E \left\{ nn^T \right\}$$

and MZ is the number of elements in the response vector.

If the values of n are uncorrelated at different time samples, the conditional probability (given the value of c) of the entire set of measured response z can be written as

$$P(z|c) = \frac{1}{(2\pi)^{N(MZ)/2} |M_1|^{N/2}} \exp \left[-\frac{1}{2} \sum_{i=1}^N (z_i - y_i)^T M_1^{-1} (z_i - y_i) \right] \quad (13)$$

The maximum conditional likelihood estimate of the unknown parameters will be the set of values of c which maximize $P(z|c)$, if it is recognized that y is a function of c . If it is noted that the maximization of $P(z|c)$ occurs for the value of c which minimizes the exponent, the maximum conditional likelihood estimate is the same as that which minimizes the mean-square response error provided the weighting matrix D_1 equals M_1^{-1} .

Maximum Unconditional Likelihood (Bayesian) Estimate

The unconditional probability of z can be expressed as

$$P(z) = P(z|c)P(c) \quad (14)$$

The probability of c relates to the a priori information available for c before use is made of the measurements z . If $P(c)$ is Gaussian,

$$P(c) = \frac{1}{(2\pi)^{(MC)/2} |M_2|^{1/2}} \exp \left[-\frac{1}{2}(c - c_0)^T M_2^{-1} (c - c_0) \right] \quad (15)$$

where

$$M_2 = E \left\{ (c - c_0)(c - c_0)^T \right\}$$

$$c_0 = E \{c\}$$

The unconditional probability of z is then

$$P(z) = \frac{1}{(2\pi)^{[(MC)+N(MZ)]/2} |M_1|^{N/2} |M_2|^{1/2}} \exp \left[-\frac{1}{2} \sum_{i=1}^N (z_i - y_i)^T M_1^{-1} (z_i - y_i) - \frac{1}{2}(c - c_0)^T M_2^{-1} (c - c_0) \right] \quad (16)$$

Again the expression is maximized by minimizing the exponent. Setting the gradient with respect to c equal to zero and solving yields

$$\hat{c}_{k+1} = \hat{c}_k + \left[\sum_{i=1}^N \nabla_c y_{i_k}^T M_1^{-1} \nabla_c y_{i_k} + M_2^{-1} \right]^{-1} \left[\sum_{i=1}^N \nabla_c^T y_{i_k} M_1^{-1} (z_i - y_{i_k}) - M_2^{-1} (\hat{c}_k - c_0) \right] \quad (17)$$

An identical estimate would have resulted if a weighted sum of mean-square response error and the mean-square difference of c and its a priori value are minimized, provided the weighting matrices used equaled the appropriate inverse error covariance matrices. Consequently, the same algorithm can be used for both estimates (ref. 11):

$$\hat{c} = \text{ARG MIN}_c \left[\sum_{i=1}^N (z_i - y_i)^T M_1^{-1} (z_i - y_i) + (c - c_0)^T M_2^{-1} (c - c_0) \right] \quad (18)$$

Computation of the Response Variable and Its Gradient

It is necessary first to solve the state equation (1) for the state variable x . The method used is that given by the recursive formula:

$$x_{i+1} = (\text{PHI})x_i + (\text{APHI})(B) \frac{u_i + u_{i+1}}{2} \quad (19)$$

where

$$\text{PHI} = e^{Ah} = I + Ah + \frac{A^2 h^2}{2!} + \frac{A^3 h^3}{3!} + \dots$$

$$\text{APHI} = \int_0^h e^{At} dt = Ih + \frac{Ah^2}{2!} + \frac{A^2 h^3}{3!} + \frac{A^3 h^4}{4!} + \dots$$

Through repeated application of the recursive formula and with the initial state and the control variable u known, it is possible to compute the state variable x . With x known, the response variable y can be computed, in turn, by using equation (2).

The method used to compute the gradient of the response variable y with respect to the unknown parameter vector c is based on an equation obtained by differentiating the state equation (1) with respect to c

$$\frac{\partial x}{\partial c} = A \frac{\partial x}{\partial c} + \frac{\partial A}{\partial c} x + \frac{\partial B}{\partial c} u \quad (20)$$

Because of the similarity of the equation and the state equation (1), it is evident that it can be solved with the recursive formula

$$\left(\frac{\partial x}{\partial c} \right)_{i+1} = (\text{PHI}) \left(\frac{\partial x}{\partial c} \right)_{i+1} + (\text{APHI}) \frac{\partial A}{\partial c} (x_i + x_{i+1}) + \frac{\partial B}{\partial c} \frac{u_i + u_{i+1}}{2} \quad (21)$$

Repeated application of the recursive formula will yield the partial derivative of the state variable with respect to c . The equation for the gradient of the response variable y with respect to c is obtained by differentiating equation (2)

$$\frac{\partial y}{\partial c} = F \frac{\partial x}{\partial c} + \frac{\partial F}{\partial c} x + \frac{\partial G}{\partial c} u + \frac{\partial b}{\partial c} \quad (22)$$

It is often the case that many of the state-variable elements are measured directly. In that event, it is very useful to compute the gradient for the first iteration by using the measured-state variable instead of the computed-state variable. This technique usually removes the need for initial estimates of the unknown parameter vector c to such a degree that a null vector for c can be used initially.

Variance of the Estimates

A very important aspect of systems identification is the quality of the estimates. Since the estimates themselves can be considered to be random numbers, it is useful to consider their variance, or specifically their error covariance matrix. The Cramèr-Rao bound (ref. 3) provides an estimate of the error covariance matrix and is identical to that developed in the following discussion.

Note that when the maximum unconditional likelihood estimate of c converges, it is necessary for the following equation to be satisfied:

$$\sum_{i=1}^N \nabla_c y_i^T M_1^{-1} (z_i - y_i) - M_2^{-1} (\hat{c} - c_0) = 0 \quad (23)$$

If c_{true} is introduced, and y_i is linearized with respect to c , it follows that

$$\sum_{i=1}^N \nabla_c y_i^T M_1^{-1} (z_i - y_{i,\text{true}} - \nabla_c y_i (\hat{c} - c_{\text{true}})) - M_2^{-1} (\hat{c} - c_{\text{true}}) - M_2^{-1} (c_{\text{true}} - c_0) = 0$$

After solving for $\hat{c} - c_{\text{true}}$,

$$\hat{c} - c_{\text{true}} = Q^{-1} \left[\sum_{i=1}^N \nabla_c y_i^T M_1^{-1} (n_i) - M_2^{-1} (c_{\text{true}} - c_0) \right]$$

where

$$Q = \left[\sum_{i=1}^N \nabla_c y_i^T M_1^{-1} \nabla_c y_i + M_2^{-1} \right]$$

$$n_i = z_i - y_{i,\text{true}}$$

Squaring and taking the expected value yields

$$\begin{aligned} \mathbf{E}\left\{\left(\hat{\mathbf{c}} - \mathbf{c}_{\text{true}}\right)\left(\hat{\mathbf{c}} - \mathbf{c}_{\text{true}}\right)^T\right\} &= -2\mathbf{Q}^{-1} \left[\sum_{i=1}^N \nabla_{\mathbf{c} \mathbf{y}_i}^T \mathbf{M}_1^{-1} \mathbf{E}\left\{ \mathbf{n}_i (\mathbf{c}_{\text{true}} - \mathbf{c}_0)^T \right\} \mathbf{M}_2^{-1} \right] \mathbf{Q}^{-1} \\ &\quad + \mathbf{Q}^{-1} \left[\sum_{i=1}^N \sum_{j=1}^N \nabla_{\mathbf{c} \mathbf{y}_i}^T \mathbf{M}_1^{-1} \mathbf{E}\left\{ \mathbf{n}_i \mathbf{n}_j^T \right\} \mathbf{M}_1^{-1} \nabla_{\mathbf{c} \mathbf{y}_j} \right] \mathbf{Q}^{-1} \\ &\quad + \mathbf{Q}^{-1} \mathbf{M}_2^{-1} \mathbf{E}\left\{ (\mathbf{c}_{\text{true}} - \mathbf{c}_0) (\mathbf{c}_{\text{true}} - \mathbf{c}_0)^T \right\} \mathbf{M}_2^{-1} \mathbf{Q}^{-1} \end{aligned}$$

Since

$$\begin{aligned} \mathbf{E}\left\{ \mathbf{n}_i (\mathbf{c}_{\text{true}} - \mathbf{c}_0)^T \right\} &= 0 \\ \mathbf{E}\left\{ \mathbf{n}_i \mathbf{n}_j^T \right\} &= \begin{cases} 0 & (i \neq j) \\ \mathbf{M}_1 & (i = j) \end{cases} \end{aligned}$$

and

$$\mathbf{E}\left\{ (\mathbf{c}_{\text{true}} - \mathbf{c}_0) (\mathbf{c}_{\text{true}} - \mathbf{c}_0)^T \right\} = \mathbf{M}_2^{-1}$$

$$\mathbf{E}\left\{ (\hat{\mathbf{c}} - \mathbf{c}_{\text{true}})(\hat{\mathbf{c}} - \mathbf{c}_{\text{true}})^T \right\} = \mathbf{Q}^{-1} \left[\sum_{i=1}^N \nabla_{\mathbf{c} \mathbf{y}_i}^T \mathbf{M}_1^{-1} \nabla_{\mathbf{c} \mathbf{y}_i} + \mathbf{M}_2^{-1} \right] \mathbf{Q}^{-1} = \mathbf{Q}^{-1} = \left[\sum_{i=1}^N \nabla_{\mathbf{c} \mathbf{y}_i}^T \mathbf{M}_1^{-1} \nabla_{\mathbf{c} \mathbf{y}_i} + \mathbf{M}_2^{-1} \right]^{-1}$$

which is the inverse that is required in obtaining the unconditional maximum-likelihood estimate of \mathbf{c} . If the a priori information is not used, \mathbf{M}_2^{-1} will be the null matrix so that

$$\mathbf{E}\left\{ (\hat{\mathbf{c}} - \mathbf{c}_{\text{true}})(\hat{\mathbf{c}} - \mathbf{c}_{\text{true}})^T \right\} = \left[\sum_{i=1}^N \nabla_{\mathbf{c} \mathbf{y}_i}^T \mathbf{M}_1^{-1} \nabla_{\mathbf{c} \mathbf{y}_i} \right]^{-1}$$

The resulting estimate of the parameter error covariance matrix is useful in assessing the quality of the estimated parameter values.

PROGRAM DESCRIPTION

The purpose of this digital computer program is to determine by means of a modified Newton-Raphson method the unknown parameters of a model used to describe a system. In order to accomplish this purpose, it is necessary to input the data, control information, starting values, and so forth. Recursive formula (17) represents the method on which the algorithm is based. Because of the large number of values involved, matrices are used throughout the program. In order to conserve memory and to facilitate changing the sizes of the arrays, a vector format has been used for the matrices. The first and second elements of the vector are the number of rows and columns, respectively. The third and fourth elements of the vector are the number of elements and type, respectively. Starting with the fifth element of the vector the elements of the matrix are entered in order starting with the first row.

For example, the matrix

$$\begin{bmatrix} 1.2 & -3.7 & 0.5 \\ 0.09 & 15.3 & 0.0 \end{bmatrix}$$

would be represented by

2., 3., 6., 1., 1.2, -3.7, 0.5, 0.09, 15.3, 0.,

Several subroutines are used to aid in the manipulation of the matrices the descriptions of which are given in appendix A.

Program Variables

The program variables are as follows:

A stability matrix of dynamic equation (1)

AP array of parameter information: (1) a priori value, (2) variance⁻¹, and
(3) scale factor

APHI integral of transition matrix, $\int_0^t e^{At_1} dt_1$

B control matrix of dynamic equation (1)

DC	incremental change in parameter vector
DUD,DUM	dummy arrays
DZN	bias in transformation equation (2)
DZ1	initial value of Y
D1S	weighting matrix for difference in response
D1	row vector of diagonal elements of matrix D1S
EE	weighting matrix for difference in parameter values
ENN	number of time points
F	state transformation matrix in equation (2)
FINK	scaling parameter to avoid overflow and underflow during matrix inversions
G	control transformation matrix in equation (2)
HH	time increment
I	general index
IG	array identification (1) A, (2) B, (3) DZ1, (4) DZN, (5) F, (6) G
II	row index
IJ	vector element index corresponding to matrix element I,J
IP	array of indices which identify array and element for AP
ITR	number of iterations
J	general index
JI	vector element index corresponding to matrix element J,I

JJ	column index
JK	parameter index
JKM	number of unknown parameters plus 1
JKMM1	number of unknown parameters
K	general index
LL	iteration index
LLM1	iteration index minus 1
MU	number of control elements
MX	number of state elements
MXP1 = MX + 1	
MZ	number of response elements
NN	number of time points
NNM1 = NN - 1	
NP	index of parameter information sets
NPM	number of parameter information sets
PHI	state transition matrix, $e^{A\Delta t}$
SU	scale factor for control
SUM	matrix of simultaneous equations
SZ	scale factor for response
TT	record length
U	control data
U1	control vector
XIA	factor applied to diagonal of sum to aid inversion when sum is nearly singular
X1,X2,XJ	state vectors

XJI gradient of response variable \mathbf{Y} with respect to parameter vector \mathbf{c}

\mathbf{Y} calculated response

\mathbf{Z} measured response

Example Problem

An example problem is considered both to aid in explaining the operation of the program and to act as a test case. The problem is that of determining certain stability derivatives and biases with regard to the response of an airplane to a known input. The equations used to compute the state are the following (eq. (19)):

$$\begin{pmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{pmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} p \\ r \\ \beta \\ \phi \end{pmatrix} + \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_0 \\ N_{\delta_a} & N_{\delta_r} & N_0 \\ Y_{\delta_a} & Y_{\delta_r} & Y_0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \delta_a \\ \delta_r \\ 1 \end{pmatrix}$$

The response variables are given by (eq. (2)):

$$\begin{pmatrix} p \\ r \\ \beta \\ \phi \\ \dot{p} \\ \dot{r} \\ \dot{\beta} \end{pmatrix} = \frac{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \end{bmatrix}}{\begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \end{bmatrix}} \begin{pmatrix} p \\ r \\ \beta \\ \phi \end{pmatrix} + \frac{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ L_{\delta_a} & L_{\delta_r} & L_0 \\ N_{\delta_a} & N_{\delta_r} & N_0 \\ Y_{\delta_a} & Y_{\delta_r} & Y_0 \end{bmatrix}}{\begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_0 \\ N_{\delta_a} & N_{\delta_r} & N_0 \\ Y_{\delta_a} & Y_{\delta_r} & Y_0 \end{bmatrix}} \begin{pmatrix} \delta_a \\ \delta_r \\ 1 \end{pmatrix} + DZN$$

where L_p , L_r , L_β , N_p , N_r , N_β , Y_p , Y_β , and Y_ϕ are coefficients of the A and F matrices and L_{δ_a} , L_{δ_r} , L_0 , N_{δ_a} , N_{δ_r} , N_0 , Y_{δ_a} , Y_{δ_r} , and Y_0 are coefficients of the B and G matrices. The unknown parameter vector c is

$$c = \begin{pmatrix} L_p \\ L_r \\ L_\beta \\ L_{\delta_a} \\ L_{\delta_r} \\ L_0 \\ N_p \\ N_r \\ N_\beta \\ N_{\delta_a} \\ N_{\delta_r} \\ N_0 \\ Y_p \\ Y_\beta \\ Y_0 \end{pmatrix}$$

where Y_ϕ , Y_{δ_a} , and Y_{δ_r} are considered to be known. The initial values of the unknown parameters are chosen to be zero and hence the A, B, F, and G matrices will be null matrices (only for convenience) except for the elements which are known. The time histories of the control inputs and responses are shown in figure 1. Since the responses are noise free except for numerical errors, the parameter values reached by using the modified Newton-Raphson algorithm (ref. 5) would be expected to be very close to their true values.

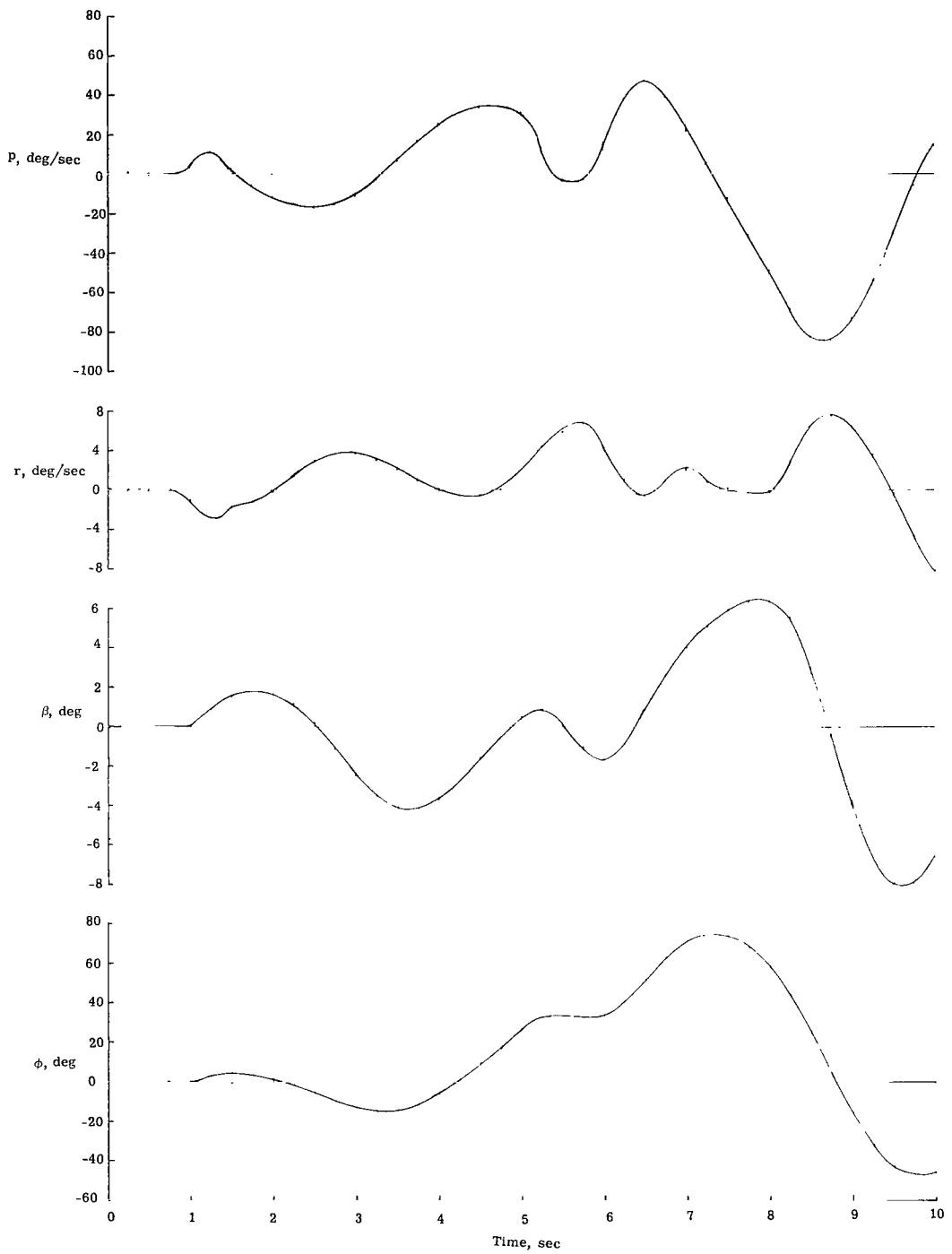


Figure 1.- Time histories of control inputs and responses used in example.

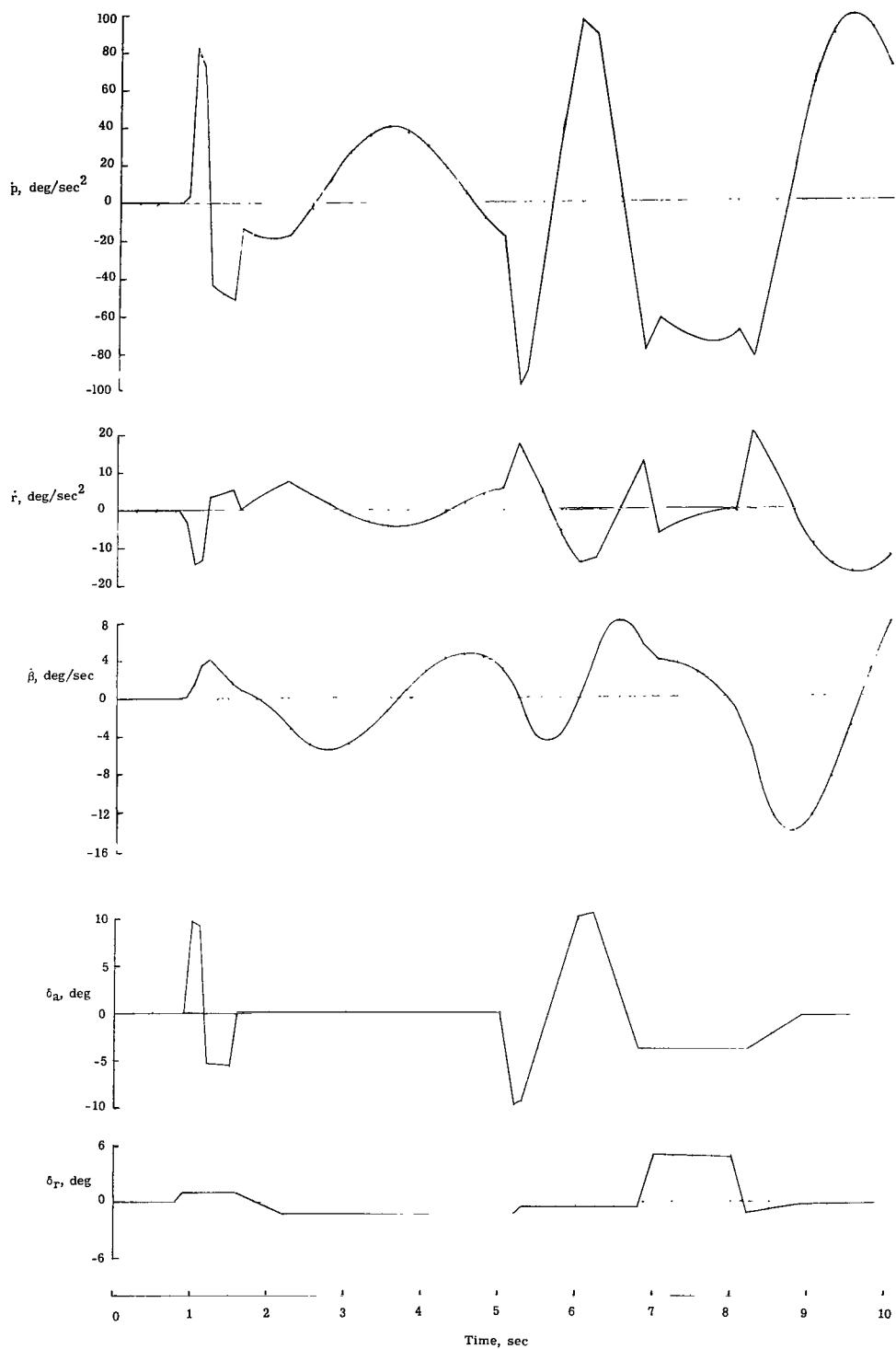


Figure 1.- Concluded.

Memory

The total storage required for the program including its subroutines is 44 547₈ fixed-point words. This requirement will change, of course, as the dimensioned arrays are changed to meet the user's specific requirements. The following expressions are offered as an aid in changing the array dimensions. The dimensions can exceed those given, of course.

A(MX · MX + 4)	G(MZ · MU + 4)
AP(NPM, 5)	IP(NPM, 4)
APHI(MX · MX + 4)	PHI(MX · MX + 4)
B(MX · MU + 4)	SU(MU + 4)
DC(JKM + 3)	SZ(MZ + 4)
DUD(MX · MX + 4)	U(MU,NN)
DUM(JKM · JKM + 4)	U1(MU + 4)
DZ1(MZ + 4)	XJ(MZ + 4)
DZN(MZ + 4)	XJI(JKM · MZ + 4)
D1(MZ + 4)	X1(MZ + 4)
D1S(MZ · MZ + 4)	X2(MZ + 4)
EE(MZ + 4)	Y(MZ + 4)
F(MZ · MX + 4)	Z(MZ,NN)

Input

The entire input for the program is in the form of data cards. It is typical, however, that the response data are recorded on magnetic tape. It will be necessary, therefore, for the user to alter the program to fit his needs. The following description of the input data cards uses an example listed in appendix B.

First card.- The first card contains two numbers: (1) NN, the number of data time points, and (2) ITR, the number of iterations. The first number must be right justified to column 10, the second to column 20.

Second card.- The second card contains a single number corresponding to HH, the time increment of the periodically sampled response of the system. The number can be positioned anywhere within the first ten columns but must contain a decimal point. The value for the scaling parameter FINK is located in the second field and can be positioned anywhere within columns 11 to 20. FINK is used to avoid overflow and underflow during matrix inversion.

Matrix format (general).- The matrices and vectors are loaded by means of subroutine load which calls for the following format. The first card contains (1) the matrix name in the first seven columns, (2) the number of rows right justified to column 10, and (3) the number of columns right justified to column 20. The next card contains the elements of the first row of the matrix with one floating point number in each of seven fields of 10 columns. Additional cards are used as necessary to complete the first row. The same procedure is used for each succeeding row with each row starting in the first field of a card.

Matrix A.- Matrix A is the systems matrix of the dynamic equation (1), the order of the coefficients can be any order that is consistent with the state variables. The order of the elements of the state vector for the example test case is p , r , β , and ϕ .

Matrix B.- The matrix B is the control matrix of the dynamic equations. The order of the coefficients of the B matrix must be consistent with that of the state and control variables. The order of the elements of the control vector for the example test case is δ_a , δ_r , and 1. The constant 1 control variable is introduced to account for possible bias in the state. This bias is not to be confused with bias in the measured response.

Matrix D1S.- The matrix D1S is the weighting matrix for differences in the response. For maximum-likelihood estimates, it should equal the inverse of the appropriate error covariance matrix. The matrix D1S can be a row vector of the diagonal elements or can be the complete square matrix. The program will recognize which you have chosen to use and will interpret D1S accordingly.

Matrix F.- The matrix F is the lower part of the transformation matrix for the state that is not an identity matrix. If the particular problem does not involve responses which are not state variables, an F matrix is not needed. In this case, indicate zero rows and zero columns on the card for the F matrix.

Matrix G.- The matrix G is the transformation matrix for the control. If the G matrix is not needed, indicate zero rows and zero columns on the card for the G matrix.

Vector SZ.- The vector SZ contains the scale factors for the response variables.

Vector SU.- The vector SU contains the scale factors for the control variables.

Array IP,AP.- Array IP,AP is a set of both fixed and floating point numbers on a single card which contains the information which relates each unknown parameter to the element of each matrix or vector involved.

(1) The first card of a set which pertains to a common parameter will have a 1 in column 10. Subsequent cards which pertain to the same parameter will have a 0 in column 10 and must follow the first card.

(2) The number in column 20 of each card identifies each array in turn involving the parameter. The code follows:

- 1 A matrix
- 2 B matrix
- 3 DZ1 vector
- 4 DZN vector
- 5 F matrix
- 6 G matrix

(3) The number in column 30 of each card identifies the row of element number in the case of a vector.

(4) The number in column 40 of each card identifies the column or is put to zero in the case of a vector.

(5) The floating-point number in columns 41 to 50 is the a priori value of the parameter. Only the first card for a particular parameter must have this information.

(6) The floating-point number in columns 51 to 60 is the standard deviation of the a priori value of the parameter. Only the first card for a particular parameter must have this information.

(7) The floating-point number in columns 61 to 70 is the scale or coefficient of the parameter for the particular element.

The first card of the IP,AP array in the input listing (appendix B) is given in the label LP. The 1 in the first field indicates the card to be the first in a group pertaining to the parameter LP. The 1, 1, and 1 in the second, third, and fourth fields indicate that the parameter is contained in the first matrix (A), first row, and first column. The large number 99999. in the next field says the variance of the a priori value of the parameter is infinite. The 1. in the next field indicates the scale or the ratio of the element to the parameter. If an a priori value was used, it would appear in the next field.

The next card has the same label LP but has a 0 or blank in its field that indicates that it is concerned with the preceding parameter. The 5, 1, and 1 indicate that the parameter is contained in the fifth matrix (F), first row, and first column, and the scale factor is 1. again. The end of the array IP,AP is indicated by a card with a negative fixed-point number in the first field of 10 columns.

Response and control data.— The response data can be read by using any mode or format that is desired. For the example test case, the data are read by using a 6E12.4 format for the variables δ_a , δ_r , 1, p, r, β , ϕ , \dot{p} , \dot{r} , and $\dot{\beta}$. Two cards are required for each of the 200 time points.

Output

The first line of output which is listed in appendix B indicates the number of data points, number of iterations, and the time increment between data points. Next, the A, B, D1, F, G, SZ, and SU matrices are listed as part of the input data. The user should check the listing of the input for errors. The A and B matrices given in the examples have all the unknown parameters set to zero initially for convenience and to demonstrate how insensitive the algorithm is to initial values of the unknown parameters.

The next block of output is IP,AP a table of the input cards used (1) to control which of the model parameters are to be adjusted, (2) to indicate which matrix elements are affected by changes in the parameter, (3) to indicate the relationship of the changes or scale, (4) to indicate the parameter's a priori value, and (5) to indicate its weighting.

The next line gives the value of JKM, the number of rows and columns of a matrix of simultaneous equations. The number of unknown model parameters to be adjusted is one less. In the same line is given the value of NPM, the number of control cards of IP,AP just described. In the example each parameter affects two matrices (A and F or B and G) so that NPM equals twice the number of unknown parameters.

The next block of output is a listing of the control and response data which is part of the input. The number of control and response variables is indicated and time is given in the first column.

The next series of blocks of output indicate the results of the repeated application of the Newton-Raphson method in solving for the unknown parameters.

First the cost or fit error is given. The cost must always be positive and should decrease and stabilize as the iterations proceed. Although it is necessary for the cost to behave in the manner just described, such behavior is not in itself sufficient evidence of a valid solution. The vector EE*D which indicates how the various components of the response error contribute to the total cost or fit error is given.

The next vector given is the root mean square of the fit error in the same units as the corresponding response variable. Although these indices are useful, it is strongly recommended that the user look at the plotted time histories to assess the quality of the fit better and to detect erroneous data values.

A vector of the partial derivatives of the cost with respect to each of the unknown parameters is also given. Theoretically, this vector should go to zero as a solution is reached. Practically, it is sufficient for the absolute value of its elements to continue to decrease as the iterations proceed. The value of the determinant of the matrix which is inverted is given in the next line. The vector DC gives the changes in the coefficients which should decrease in amplitude as a solution is approached. The last several blocks of output listed in appendix B are repeated for each iteration until the time history which

corresponds to the last coefficient values given is listed. Next is given the regular information for the last iteration followed by the A, B, DZ1, DZN, F, and G matrices.

The last array in the output provides a summary of the results, including the final estimate and the approximate standard deviation if the inverse of the noise covariance matrix is used for D1S. The a priori values and a priori standard deviation are provided for comparison.

Trouble Shooting

Although there is no replacement for the investigator's understanding of his particular problem, it is useful to point out some of the more common difficulties encountered in trying to obtain a solution.

Case 1.- In case 1, the fit error converges as do most of the parameters but a few parameters diverge. Perhaps the most common difficulty in systems identification is caused by trying to determine a greater number of parameters than are justified by the amount and quality of the measured data. The only solution is to (1) increase the information processed by adding to the data base or adding a priori information, (2) ask for fewer parameters to be identified, or (3) improve the quality of the data by increasing the signal-noise ratio or by using more suitable inputs which reduce the correlation between variables. Determine that the same parameter does not occur twice in the control cards.

Case 2.- In case 2 an error message, "Matrices Incompatible for MULT,DUM-DUM . . ." is listed repeatedly. The problem is just as the error message says and is caused by inconsistent input matrices. The number of states MX is determined by the number of rows in A. Matrix B must have the same number of rows; matrices A and F must have MX columns. The number of controls MU is determined by the number of columns in B. Matrix G must have the same number of columns. The number of responses MZ is determined by the number of columns in D1. Matrices F and G must have MZ - MX rows. The matrices listed immediately after the error message are those for which multiplication was attempted.

Case 3.- In case 3, the fit error neither converges nor diverges but just drifts. Something is wrong. Check to see that the data is in the proper order and that the parameter control cards are correct. Also, a model form which is in error could cause this type of behavior.

Case 4.- In case 4, the fit error converges to a point and then fluctuates wildly. In certain marginal cases the numerical errors associated with inverting a matrix which is nearly singular can cause the fit error and parameter values to fluctuate. This problem can be alleviated by (1) reducing the number of unknown parameters, (2) adding a priori information, or (3) increasing slightly the factor multiplying the diagonal of the SUM

matrix. Care must be taken if the third option is chosen since a factor too large will make the algorithm appear to converge when it is simply sluggish.

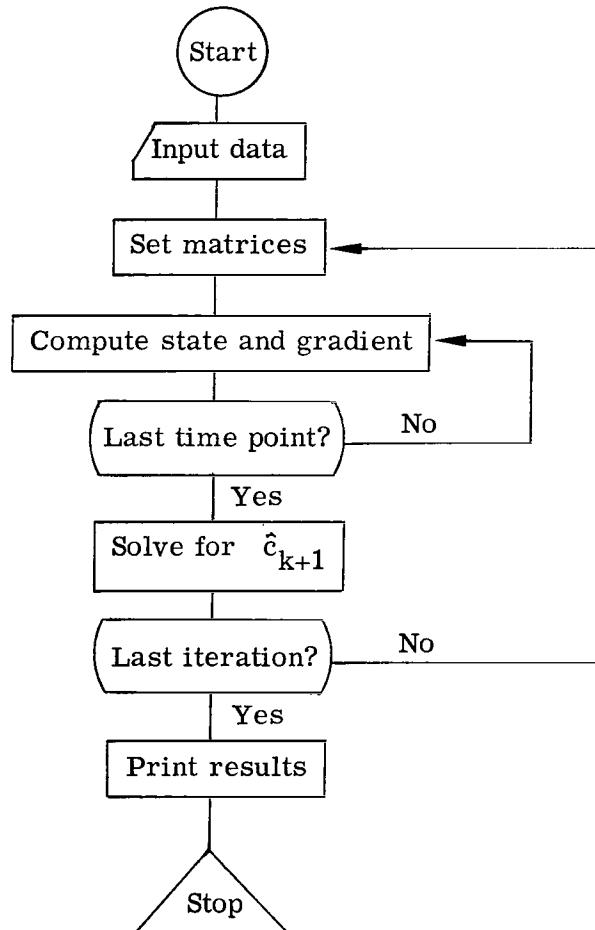
Case 5.- In case 5, the fit-error parameter values converge and the gradient continues to diminish. These characteristics indicate that a bona fide solution has been reached. Another indication of the quality of the estimate of the parameter is its standard deviation. If the standard deviation of the parameter is not noticeably smaller than its original value, the additional information contained in the measured data did little to improve the estimate of its value.

Program NEWTON

The program NEWTON performs system identification in state space format by using a modified Newton-Raphson method. Based on the recursive relationship,

$$\hat{c}_{k+1} = \hat{c}_k + \left[\sum_{i=1}^N \nabla_c y_{i,k}^T M_1^{-1} \nabla_c y_{i,k} + M_2^{-1} \right]^{-1} \left[\sum_{i=1}^N \nabla_c^T y_{i,k} M_1^{-1} (z_i - y_{i,k}) - M_2^{-1} (\hat{c}_k - c_0) \right]$$

The flow chart follows:



The listing for program NEWTON is

```

PROGRAM NEWTON (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
.....SYSTEM IDENTIFICATION PROGRAM....STABILITY DERIVATIVES FROM FLIGHT DATA..
DIMENSION Z(7,400), U(5,400), DZ1(12),DZN(12),IP(60,4), AP(60,5)
1,EE(12), DUD(68), APHI(68), X1(16), X2(16), XJ(16), U1(12),
2PHI(68), D1(12), A(68), B(36), XJI(733), SUM(733),DC(30),DUM(733)
3, F(60), G(60), SZ(25), SU(25), Y(16),DIS(68)
111 FORMAT(7X, I3,3I10,5F10.4)
606 FORMAT(6E12.4)
700 FORMAT(8F10.4)
777 FORMAT(7I10)
1001 FORMAT(1IH THERE ARE ,I4,14H DATA POINTS, ,I2,38H ITERATIONS AND T
1HE TIME INCREMENT IS ,F5.4,8H SECONDS)
1002 FORMAT(8OH      PARAMETER GROUP      ROW      COLUMN APRIORI WEIG
1HT      SCALE      ANSWER )
1003 FORMAT(6H JKM =,I3,7H, NPM =,I3)
1004 FORMAT(36H THE FIRST COLUMN IS TIME. THE NEXT ,I2,41H COLUMNS ARE
1CONTROL VARIABLES. THE NEXT ,I2,32H COLUMNS ARE RESPONSE VARIABLE
2S.)
1005 FORMAT(31H THE VALUE OF THE FIT ERROR IS ,E12.5,26H FOR THE ITERAT
1ION NUMBER ,I3)
1006 FORMAT(39H THE SCALING PARAMETER, FINK#S VALUE IS,F10.4)
1010 FORMAT(F8.2,10E12.4)
CIIIIIIIIIIIIIIIIIIIIIIIIIIII.....INPUT DATA (START)....$3
3 READ 777, NN, ITR
READ 700, FH, FINK
PRINT 1001, NN, ITR, HH
PRINT 1006, FINK
ENN=NN
CALL LOAD(4, A, B, DIS, F)
CALL LOAD(3, G, SZ, SU, SU)
CALL SPIT(A, 1HA)
CALL SPIT(B, 1HB)
CALL SPIT(DIS, 3HDIS)
CALL SPIT(F, 1HF)
CALL SPIT(G, 1HG)
CALL SPIT(SZ, 2HSZ)
CALL SPIT(SU, 2HSU)
NNM1 = NN - 1
MU = B(2) + .01
MX = A(2) + .01
MZ = DIS(2) + .01
MXPI = MX + 1
IB = DIS(1) + .01
IF(IB-1)78,79,78
78 CALL SET(D1, 1,MZ)
IJ = 4 - MZ
DO 80 I = 1,MZ
IJ = IJ + MZ + 1
80 D1(I+4) = DIS(IJ)
GO TO 81
79 CALL MAKE(C1, DIS)
81 CALL SPIT(C1, 2FC1)
AD = 1.0/ENN
CALL ADD(AD, D1, 0., D1, D1)
CALL ADD(AD, DIS, 0., DIS, DIS)
PRINT 1002
JKM = 1
DO 5 NP = 1,60
READ 111,(IP(NP,I), I = 1,4),(AP(NP,J), J = 1,3)
PRINT111,(IP(NP,I), I = 1,4),(AP(NP,J), J = 1,3)
AP(NP,4) = AP(NP,1)
AP(NP,5) = 0.
IF(IP(NP,1))6,5,5
5 JKM = JKM + IP(NP,1)

```

```

6 CONTINUE
JKMM1 = JKM - 1
NPM = NP - 1
PRINT 1003, JKM, NPM
TT = - HH
PRINT 1004, MU, MZ
DO 48 I = 1,NN
READ 606,(U(K,I), K = 1,MU),(Z(K,I), K = 1,MZ)
TT = TT + HH
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC....CONDITION INPUT DATA (START)...
DO 63 J = 1,MZ
63 Z(J,I) = Z(J,I)*SZ(J+4)
DO 64 J = 1,MU
64 U(J,I) = U(J,I)*SL(J+4)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC....CONDITION INPUT DATA (END)....
48 PRINT1010,TT,(U(K,I), K = 1,MU),(Z(K,I), K = 1,MZ)
CIIIIIIIIIIIIIIIIIIIIIIIIIIII....INPUT DATA (END)....$  

CLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL....ITERATION LOOP (START)....$  

DO 12 LL=1,ITR
CSSSSSSSSSSSSSSSSSSSSSSSSSSSS....SET MATRICES (START)....$  

CALL SET(U1, MU, 1)
CALL SET(X1, MX, 1)
CALL SET(XJ,MX,1)
CALL SET(Y, MZ, 1)
CALL SET(EE,1,MZ)
CALL SET(DZ1,1,MZ)
CALL SET(DZN,1,MZ)
CALL SET(SUM, JKM, JKM)
CALL SET(XJI, JKM, MZ)
CSSSSSSSSSSSSSSSSSSSSSSSSSS....SET MATRICES (END)....$  

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC....COMPUTE STATE TRANSITION MATRIX.
CALL EAT(A, HH, PHI, APHI, DUM, DUD)
JK = 0
DO 10 NP = 1,NPM
IG = IP(NP,2)
JJ = IP(NP,3)
IF(IP(NP,1)-1)71,70,71
70 JK = JK + 1
II = (JK-1)*JKM + JK + 4
SUM(II) = 1./AP(NP,2)**2
71 IF(IP(NP,2) - 3)10,16,16
16 IF(IG-5)8,10,10
8 JI = (JK-1)*MZ + 4 + IP(NP,3)
XJI(JI) = AP(NP,3)
IF(LL-1)10,55,10
55 IF(IP(NP,2)-3)10,17,20
17 DZ1(4+JJ) = AP(NP,1)*AP(NP,3)
GO TO 10
20 DZN(4+JJ) = AP(NP,1)*AP(NP,3)
10 CONTINUE
DO 51 I = 1,MU
51 U1(I+4) = (U(I,1) + U(I,2))*.5
DO 24 J = 1,MX
24 X1(J+4) = Z(J,1) - DZ1(J+4) - DZN(J+4)
C11111111111111111111111111111111....COMPUTE INITIAL STATE(START)...
CALL MULT(PHI, X1, X2)
CALL MULT(B, U1, DUM)
CALL MULT(APHI, DUM, DUD)
CALL ADD(1.0,X2,1.0,DUD,X2)
DO 47 I = 1,MX
47 Y(I+4) = X2(I+4)
TT = 0.
C11111111111111111111111111111111....COMPUTE INITIAL STATE(END)....$  

CTTTTTTTTTTTTTTTTTTTTTTTTTTTTT....TIME LOOP (START)....$  


```



```

68 IF(I-NNM1)72,41,72
72 CALL MAKE(X1, X2)
DO 30 J = 1,MU
30 U1(J+4) = (U(J,I+1) + U(J,I+2))*5
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS....COMPUTE STATE (START).....$  

CALL MULT(PHI, X1, X2)
CALL MULT(B, U1, DUD)
CALL MULT(APHI, DUD, DUM)
CALL ADD(1.0,X2,1.C,DUM,X2)
DO 2 J = 1,MX
2 Y(J+4) = X2(J+4)
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS....COMPUTE STATE (END).....$  

CEEEEEEEEEE.....SET UP SYSTEM OF EQUATIONS.....$  

41 CALL TRICK(XJI, DIS, SUM)
CTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTT....TIME LOOP (END).....$  

LLM1 = LL-1
PRINT 1005, SUM(JKM*JKM+4),LLM1
CALL SPIT(EE,4HEE*D)
DO 60 I = 1,MZ
60 EE(I+4) = SQRT(EE(I+4)/(D1(I+4)*ENN))/SZ(I+4)
CALL SPIT(EE, 3HRMS)
CDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD...MULT DIAG BY 1.0005 TO CONDITION.$  

CALL SET(DUM, 1, JKM)
DO 1 J = 1,JKM
JJ = JKM*J + 4
IJ = (J-1)*JKM + J + 4
DUM(J+4) = SUM(JJ)
1 SUM(IJ) = SUM(IJ)*1.0005
CALL SPIT(DUM, 4HGRAD)
531 FORMAT(55H WOULD YOU BELIEVE.....)
PRINT 531
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS....SOLVE SYSTEM OF EQUATIONS.....$  

CALL ADD(FINK, SUM, 0., SUM, SUM)
IF(LL-ITR)74,73,74
73 CALL MINR(SUM, JKM, JKM, XJI)
CALL SET(DUD, JKMM1, 1)
JJ = 4
DO 75 J = 1,JKMM1
JJ = JJ + JKM
75 DUD(J+4) = SUM(JJ)
74 CALL SOLVE(SUM, DC)
CALL SPIT(DC, 2HDC)
CUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUU....UPDATE COEFFICIENTS (START)....$  

JK = 0
DO 12 NP = 1,NPM
IG = IP(NP,2)
II = IP(NP,3)
JJ = IP(NP,4)
JK = JK + IP(NP,1)
IF(JK)12,12,50
50 IF(IP(NP,1)-1)57,56,57
56 AP(NP,4) = AP(NP,4) + DC(JK+4)
57 IF(IG-1)12,19,21
19 IJ = (II-1)*MX + JJ + 4
A(IJ) = A(IJ) + DC(JK+4)*AP(NP,3)
GO TO 12
21 IF(IG-2)12,22,37
22 IJ = (II-1)*MU + JJ + 4
B(IJ) = B(IJ) + DC(JK+4)*AP(NP,3)
37 IF(IG-3)12,36,38
36 DZ1(IJ+4) = DZ1(IJ+4) + DC(JK+4)*AP(NP,3)

```

CONCLUDING REMARKS

A FORTRAN program has been developed which computes maximum-likelihood estimates or determines the parameters of a linear model which minimize the weighted mean-square difference between the model and the measured responses. The responses can be any linear transformation of the state and control variables. A priori information can be included and the initial state values and biases can also be determined. Several different parameter estimates are described and shown to be either identical or similar. An estimate of the error covariance matrix of the estimated parameters which is identical to the Cramèr-Rao bound is derived.

The program is described to enable the prospective user to apply the method to his particular problem after first running the check case provided. A few trouble-shooting tips are given as well. Copies of the program and test case will be provided by the author upon request.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., March 20, 1972.

APPENDIX A

SUBROUTINES USED IN PROGRAM

The subroutines described in this appendix are used in the main program NEWTON for a variety of matrix operations. In most cases the operation is simply the name of the subroutine. Since the matrices have vector format and all arrays of the subroutines appear in the call statement, all dimensioning of the arrays is done in the main program. The subroutines have a variety of uses besides the program discussed. The user is cautioned that little error checking is performed beyond checking the matrices' compatibility.

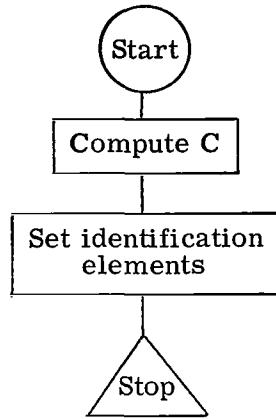
APPENDIX A — Continued

Subroutine ADD(P,A,Q,B,C)

Purpose: To add two matrices or vectors after each has been multiplied by a scalar

Formula: $C = PA + QB$

Flow chart:



Description: Subroutine ADD can operate on two compatible matrices in the vector format to (1) add, (2) subtract, (3) multiply by a scalar, or (4) perform some combination of these operations.

Program variables:

A	first matrix or vector
B	second matrix or vector
C	weighted sum or difference of A and B
P	first scalar
Q	second scalar

Subroutine listing:

```
SUBROUTINE ADD(P, A, Q, B, C)
C      VECTOR FORMAT
      DIMENSION A(1), B(1), C(1)
      N = A(3) + 4.01
      DO 1 I = 5,N
1      C(I) = P*A(I) + Q*B(I)
      C(1) = A(1)
      C(2) = A(2)
      C(3) = A(3)
      C(4) = A(4)
      RETURN
      END
```

APPENDIX A – Continued

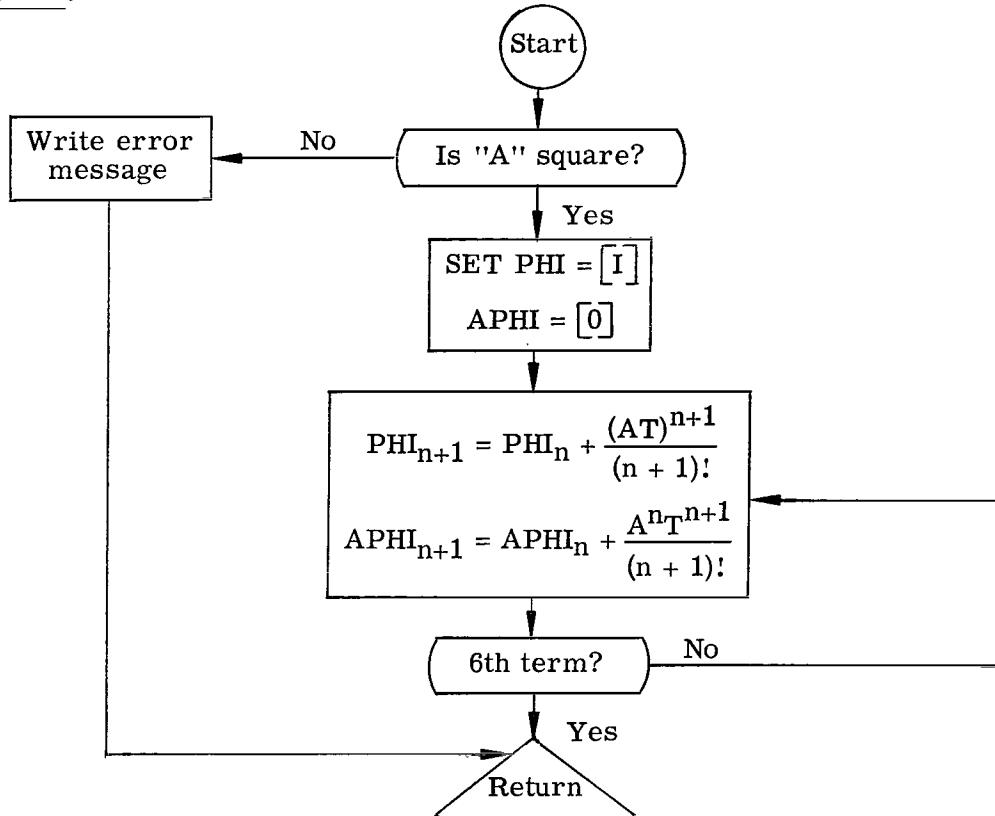
Subroutine EAT(A,T,PHI,APHI,A2,A3)

Purpose: To form a state transition matrix and its integral for use in computing time histories

Formula: $\text{PHI} = \sum_{n=0}^{6} \frac{(AT)^n}{n!} \approx e^{AT}$

$$\text{APHI} = \sum_{n=1}^{6} \frac{A^{n-1}T^n}{n!} \approx \int_0^T e^{AT} dt$$

Flow chart:



Description: Subroutine EAT employs a finite power series to approximate the state transition matrix PHI and its integral APHI. The state is calculated by using the following formula:

$$X_{i+1} = (\text{PHI})X_i + (\text{APHI})(B) \frac{u_{i+1} + u_i}{2}$$

APPENDIX A – Continued

The subroutine uses six terms of the power series but can be easily modified to include as many as is necessary. Since it is also assumed that the control input is constant over the time interval, a more accurate method of integration may be necessary in some cases.

<u>Program variables:</u>	A	stability matrix of $\dot{x} = Ax + Bu$
	A2	dummy matrix
	A3	dummy matrix
	APHI	integral of state transition matrix, $\int_0^T e^{AT} dt$
	BB	floating-point variable equal to term index
	G	scalar coefficient in power series
	I	general index
	II	number of rows of A and therefore of PHI and APHI
	JJ	number of columns of A and must be equal to II
	PHI	state transition matrix, e^{AT}
	T	time increment over which transition matrix is desired. The value of T must be kept small compared with shortest time constant of system and cannot in gen- eral, be made the running time.

Subroutine listing:

```

      SUBROUTINE EAT(A, T, PHI, APHI, A2, A3)
C   VECTOR FORMAT
      DIMENSION A(1), PHI(1), APHI(1), A2(1), A3(1)
123 FORMAT(55H MATRIX IS NOT SQUARE FOR EAT, STUPID.....
      II = A(1) + .01
      JJ = A(2) + .01
      IF(II-JJ)4,3,4
      3 CALL SET(PHI, II, II)
      CALL SET(APHI, II, II)
      DO 1 I = 1,II
      IJ = (I-1)*II + I + 4
      1 PHI(IJ) = 1.0
      CALL MAKE(A2, PHI)
      G = 1.0
      DO 2 I = 1,6
      BB = I
      G = G*T/BB
      CALL ADD(1.0, APHI, G, A2, APHI)
      CALL MULT(A, A2, A3)
      CALL MAKE(A2, A3)
      CALL ADD(1., PHI, G, A2, PHI)
      2 CONTINUE
      GO TO 5
      4 PRINT 123
      5 RETURN
      END

```

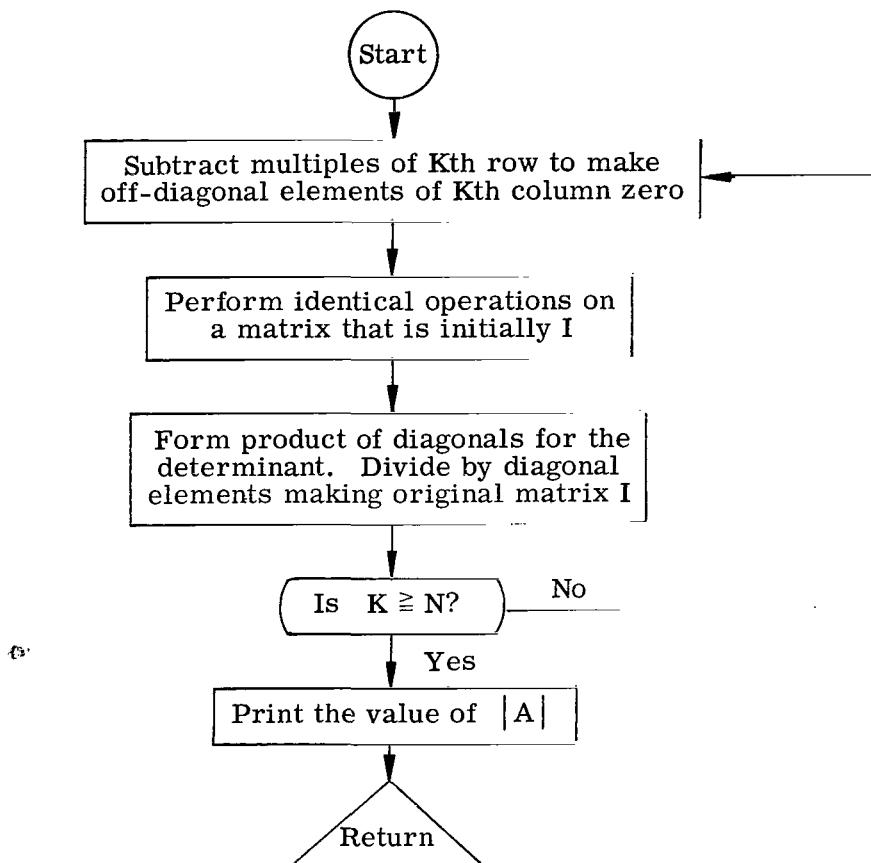
APPENDIX A – Continued

Subroutine INVR(A,B)

Purpose: To invert a matrix

Formula: $B = A^{-1}$

Flow chart:



Description: Subroutine INVR inverts the matrix A by using a Gauss-Jordan pivotal method in which the matrix A is diagonalized by subtracting multiples of succeeding rows to make off-diagonal elements zero. Identical manipulations are made on a matrix which is initially an identity matrix the size of A. The diagonal elements of the diagonalized matrix are then used as divisors for the corresponding rows of the other matrix and thus make the latter matrix A^{-1} . The determinant of A is also determined and printed.

<u>Program variables:</u>	A	matrix (square) to be inverted
	B	inverse of A
	DETA	determinant of A

APPENDIX A – Continued

I	row index
J	column index
K	pivot index
IJ,KPI,KK, KIX,KL,KIL}	general indices
AKK1,AKIK	general variables

Accuracy: $\left| \frac{A^{-1}A - I}{\text{off-diagonal elements}} \right| < 10^{-10}$ (N = 15)

(Accuracy, of course, is problem dependent. The value given reflects the author's experience on the Control Data 6600 computer with single-precision arithmetic.)

Subroutine listing:

```

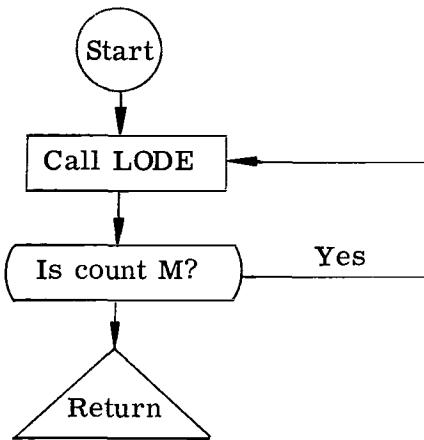
SUBROUTINE INVR(A, B)
C   VECTOR FORMAT, DIAGONAL ELEMENTS MUST BE NON-ZERO.
DIMENSION A(1), B(1)
101 FORMAT(E15.7)
      N = A(1) + .01
      CALL SET(B, N, N)
      DO 6 I = 1,N
      IJ = (I-1)*N + I + 4
      6 B(IJ) = 1.0
      DETA = 1.0
      DO 3 K = 1,N
      KP1 = K + 1
      KK = (K-1)*N + K + 4
      AKK1 = 1.0/A(KK)
      DETA = DETA*A(KK)
      DO 5 J = 1,N
      IJ = (K-1)*N + J + 4
      5 B(IJ) = B(IJ)*AKK1
      DO 1 J = K,N
      IJ = (K-1)*N + J + 4
      1 A(IJ) = A(IJ)*AKK1
      DO 2 KI = 1,N
      IF(K - KI)7,2,7
      7 KIK = (KI-1)*N + K + 4
      AKIK = A(KIK)
      DO 4 L = 1,N
      KL = (K-1)*N + L + 4
      KIL = (KI-1)*N + L + 4
      B(KIL) = B(KIL) - AKIK*B(KL)
      4 CONTINUE
      IF(K-N)8,9,8
      8 DO 10 L = KP1,N
      KL = (K-1)*N + L + 4
      KIL = (KI-1)*N + L + 4
      A(KIL) = A(KIL) - AKIK*A(KL)
      10 CONTINUE
      9 CONTINUE
      2 CONTINUE
      3 CONTINUE
      PRINT 101, DETA
      RETURN
      END

```

APPENDIX A – Continued
Subroutine LOAD(M,A,B,C,D)

Purpose: To read input cards of M arrays (matrices or vectors)

Flow chart:



Description: Subroutine LOAD inputs up to four matrices through the repeated use of subroutine LODE. All arrays are put into the vector format described previously.

Program variables:

A	first array
B	second array
C	third array
D	fourth array
M	number of arrays

Subroutine listing:

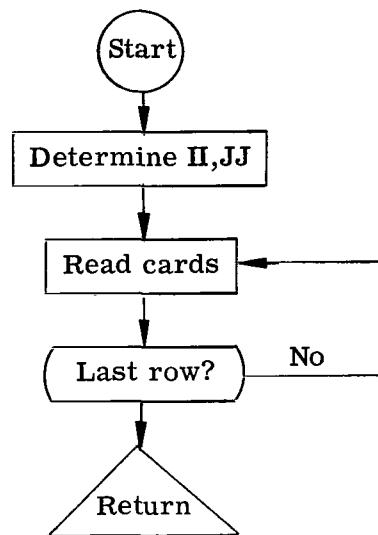
```
SUBROUTINE LOAD(M, A, B, C, D)
C   VECTOR FORMAT
      DIMENSION A(1), B(1), C(1), D(1)
      CALL LODE(A)
      IF(M-1)8,8,2
      2 CALL LODE(B)
      IF(M-2)8,8,4
      4 CALL LODE(C)
      IF(M-3)8,8,6
      6 CALL LODE(D)
      8 RETURN
      END
```

APPENDIX A – Continued

Subroutine LODE(A)

Purpose: To read input cards of a single array (matrix or vector)

Flow chart:



Description: Subroutine LODE reads the first card to determine the number of rows and columns of the array. The required number of cards are read for each row until all rows are read. The array is put into the vector format previously described.

Program variables:

A	input array
I	row index
II	number of rows
J	column index
JJ	number of columns
L	general index

APPENDIX A - Continued

Subroutine listing:

```
SUBROUTINE LODE(A)
C   VECTOR FORMAT
      DIMENSION A(1)
 700 FORMAT(7F10.3)
 444 FORMAT(7X,I3,7X,I3)
      READ 444, II, JJ
      A(1) = II
      A(2) = JJ
      A(3) = II*JJ
      A(4) = 0.
      IF(A(3).GT.3,2)
 2 CONTINUE
      L = 4 - JJ
      DO 1 I = 1,II
      L = L + JJ
 1 READ 700,(A(L+J), J = 1,JJ)
 3 CONTINUE
      RETURN
      END
```

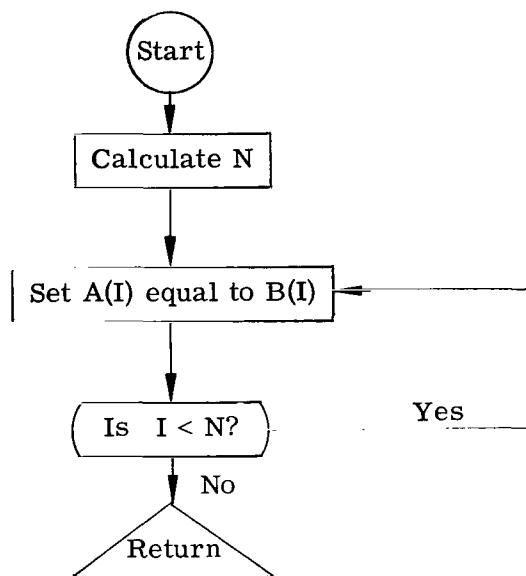
APPENDIX A – Continued

Subroutine MAKE(A,B)

Purpose: To make array A equal to array B

Formula: $A = B$

Flow chart:



Description: Subroutine MAKE sets all the elements of array A equal to those of array B, including the elements which indicate the size of the array. The vector format is used as previously described.

Program variables: A new array

B original array

I general index

N number of elements including those giving number of rows,
columns, and so forth

Subroutine listing:

```
SUBROUTINE MAKE(A, B)
C   VECTOR FORMAT
      DIMENSION A(1), B(1)
      N = B(1)*B(2) + 4.01
      DO 1 I = 1,N
1   A(I) = B(I)
      RETURN
      END
```

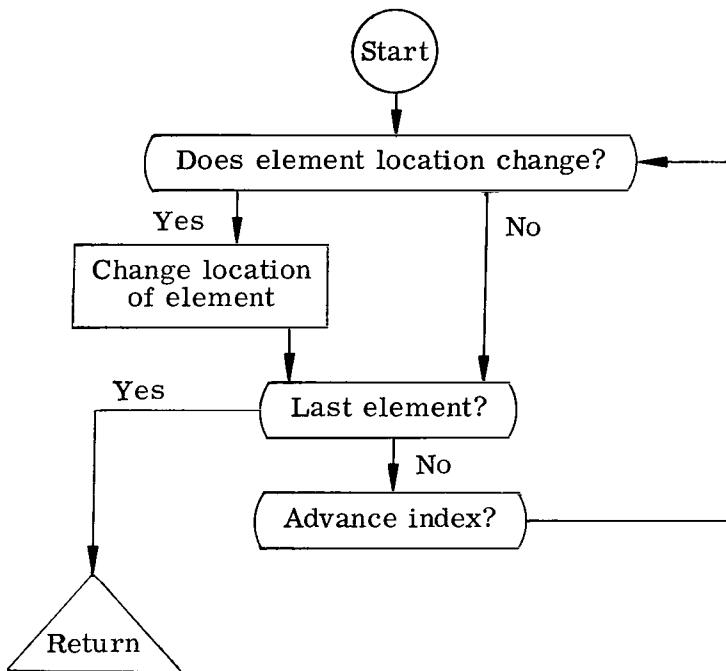
APPENDIX A – Continued

Subroutine MINR(A,I,J,B)

Purpose: To form the minor of matrix A

Formula: $B = \text{MINOR}_{I,J}[A]$

Flow chart:



Description: The minor of matrix A is formed by eliminating the Ith row and Jth column.

Program variables:

A	matrix for which minor is sought
B	resulting minor matrix
I	row to be eliminated
J	column to be eliminated
K,L,II,JJ	general indices

APPENDIX A – Continued

Subroutine listing:

```
SUBROUTINE MINR(A, I, J, B)
C   VECTOR FORMAT
      DIMENSION A(1), B(1)
      M = A(1) + .01
      N = A(2) + .01
      IF(I)11,10,11
10  I = 99999
      MM1 = M
      GO TO 12
11  MM1 = M - 1
12  IF(J)13,14,13
14  J = 99999
      NM1 = N
      GO TO 9
13  NM1 = N - 1
9   DO 1 K = 1,M
     DO 1 L = 1,N
     IF(I-K)2,8,3
2   II = K - 1
     GO TO 4
3   II = K
4   IF(J-L)5,8,6
5   JJ = L-1
     GO TO 7
6   JJ = L
7   B(II*NM1 + JJ - NM1 + 4) = A(K*N + L - N + 4)
8   CONTINUE
1   CONTINUE
      B(1) = MM1
      B(2) = NM1
      B(3) = MM1*NM1
      B(4) = 0.
      RETURN
      END
```

APPENDIX A – Continued

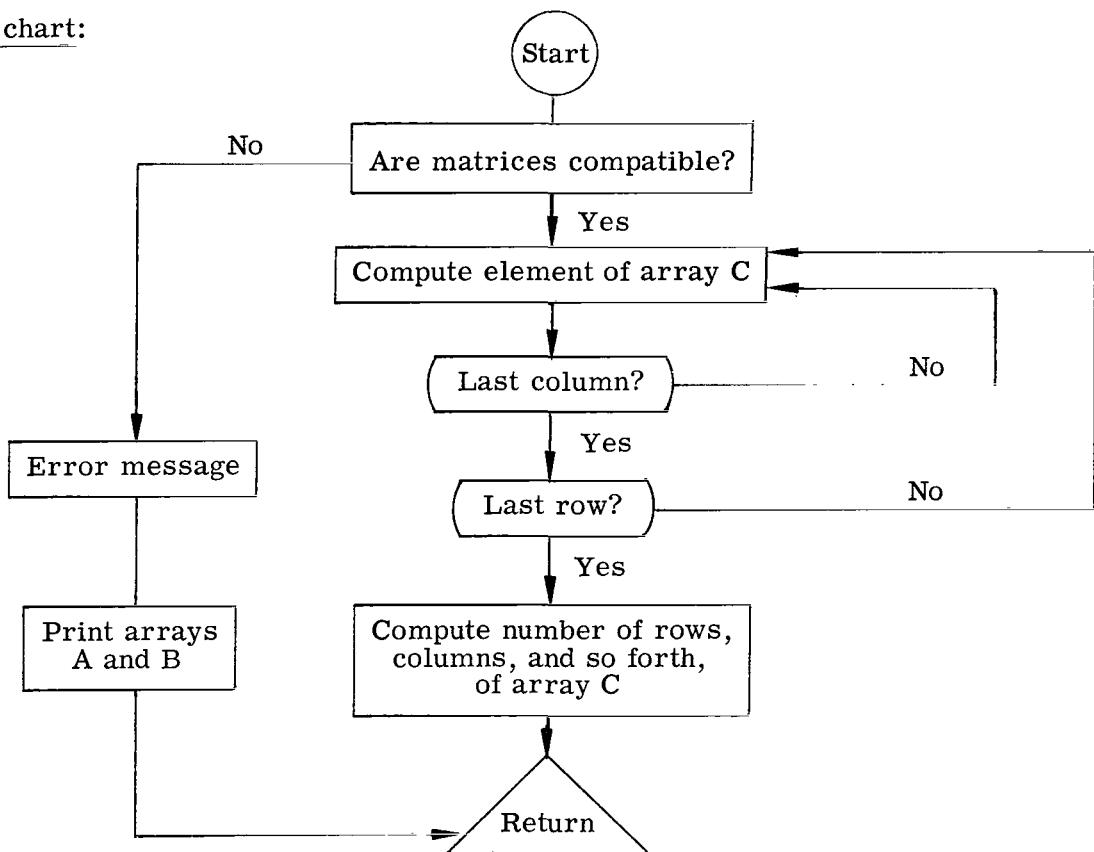
Subroutine MULT(A,B,C)

Purpose: To compute the product of two arrays (matrices or vectors)

Formula: $C = AB$

$$C_{ij} = \sum_{k=1}^{KK} a_{ik} b_{kj}$$

Flow chart:



Description: Subroutine MULT generates an array C which is the product of arrays A and B. Since the elements of C are changed during the operation, array C cannot be the same as either array A or B.

<u>Program variables:</u>	A	first array in product
	B	second array in product
	C	product array

APPENDIX A – Continued

I	row index
II	number of rows in A
J	column index
JJ	number of columns in B
KK	number of columns in A
LL	number of rows in B
IJ,IK,KJ	general indices

Subroutine listing:

```

      SUBROUTINE MULT(A, B, C)
C   VECTOR FORMAT - NEITHER A NOR B CAN BE EQUIVALENT TO C
      DIMENSION A(1), B(1), C(1)
      II = A(1) + .01
      JJ = B(2) + .01
      KK = A(2) + .01
      LL = B(1) + .01
      CALL SET(C, II, JJ)
      IF(KK - LL)12,11,12
12 PRINT 123
123 FORMAT(55H MATRICES INCOMPATIBLE FOR MULT, DUM-DUM.... )
11 DO 1 I = 1,II
    DO 1 J = 1,JJ
    DO 1 K = 1,KK
      IJ = (I-1)*JJ + J + 4
      IK = (I-1)*KK + K + 4
      KJ = (K-1)*JJ + J + 4
1 C(IJ) = C(IJ) + A(IK)*B(KJ)
1 C(1) = A(1)
1 C(2) = B(2)
1 C(3) = II*JJ
1 C(4) = 0.
      RETURN
      END

```

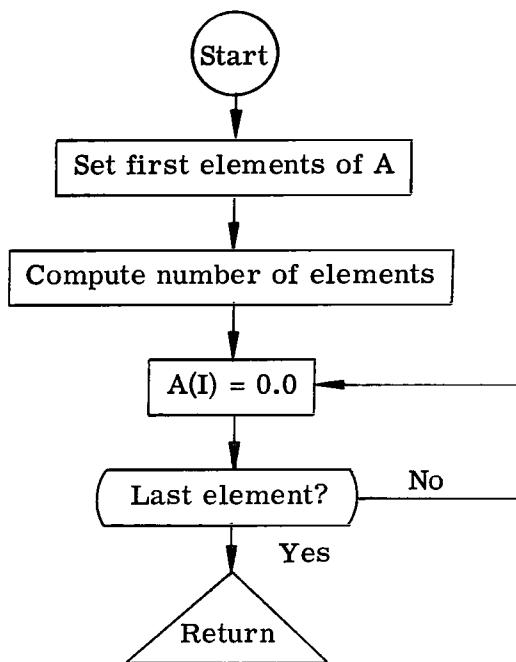
APPENDIX A – Continued

Subroutine SET(A,II,JJ)

Purpose: To generate an array with all its elements set to zero

Formula: $A = [0]$

Flow chart:



Description: Subroutine SET generates a null array of desired size. The array must still be dimensioned although it originates by calling the subroutine. A vector format is used as was described earlier.

Program variables: A null array generated

II number of rows

JJ number of columns

N total number of elements including special elements

Subroutine listing:

```

C      SUBROUTINE SET(A, II, JJ)
      VECTOR FORMAT
      DIMENSION A(1)
      A(1) = II
      A(2) = JJ
      A(3) = II*JJ
      A(4) = 0.
      N = II*JJ + 4
      DO 1 I = 5,N
1   A(I) = 0.0
      RETURN
      END
  
```

APPENDIX A – Continued

Subroutine SOLVE(A,C)

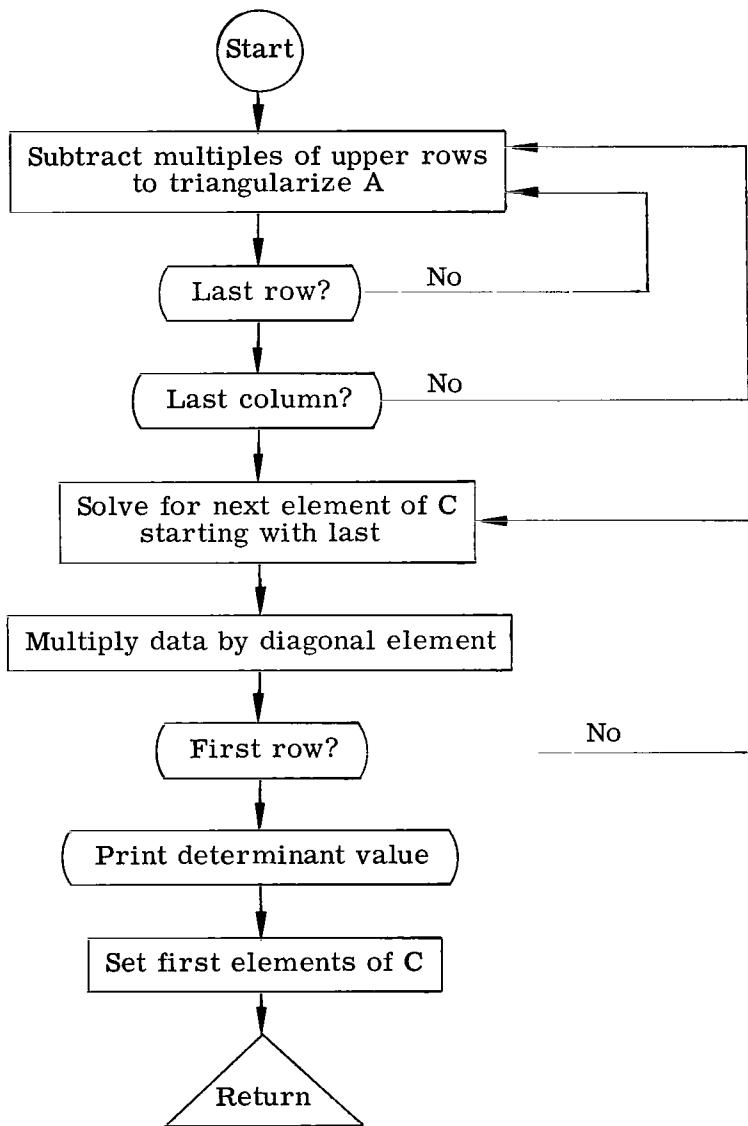
Purpose: To solve a linear system of algebraic equations $\mathbf{Y} = \mathbf{BC}$

Formula: $\mathbf{C} = \mathbf{B}^{-1}\mathbf{Y}$

where

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{B} & \mathbf{Y} \\ \hline - & - \end{array} \right]$$

Flow chart:



APPENDIX A – Continued

Description: Subroutine SOLVE employs Gauss' pivotal or elimination method to first triangularize the matrix B and then solve for the elements of the solution vector C in reverse order. The product of the diagonal terms of the triangularized matrix is the determinant, the value of which is printed to indicate a proper range of amplitudes of numbers involved. The matrix B consists of A except for the right-hand column which is Y. Both matrix A and vector C use the vector format described earlier.

<u>Program variables:</u>	A	juxtaposition of matrix of coefficients of simultaneous equations B and constant vector Y
	C	solution vector
	DETA	determinant
	N	number of unknowns, simultaneous equations

Subroutine listing:

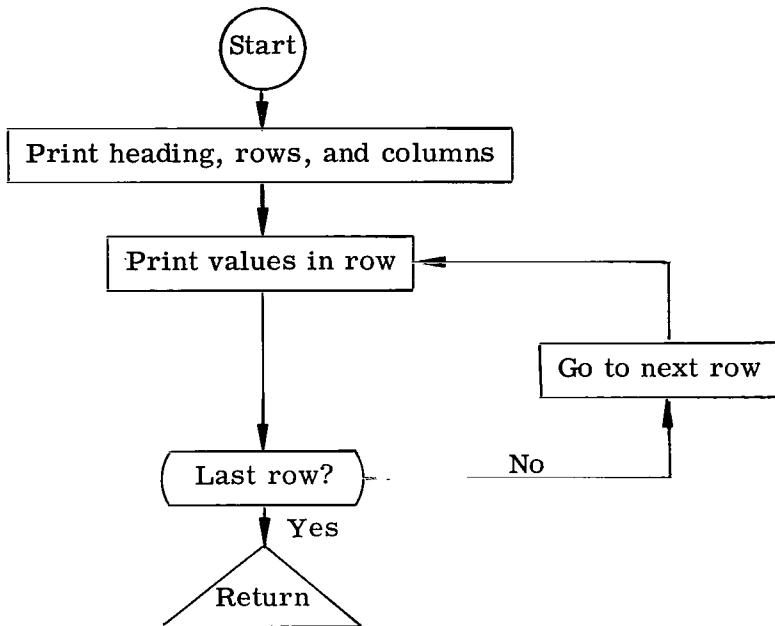
```
SUBROUTINE SOLVE(A, C)
C   VECTOR FORMAT
      DIMENSION A(1), C(1)
101 FORMAT(E15.7)
      N = A(1) - .99
      NM1 = N - 1
      NP1 = N + 1
      NP3 = N - 3
      DO 6 K = 1,NM1
      AKK1 = 1./A(K*NP1 + K - NP3)
      K1 = K + 1
      DO 6 KI = K1,N
      AKIK = A(KI*NP1 + K - NP3)*AKK1
      DO 6 L = K1,NP1
      KIL = KI*NP1 + L - NP3
6     A(KIL) = A(KIL) - AKIK*A(K*NP1 + L - NP3)
      DETA = A(5)
      C(N+4) = A(NP1*NP1 - NP3)/A(N*NP1 + 3)
      DO 16 I = 2,N
      NI1 = N - I + 1
      C(NI1 + 4) = A(NI1*NP1 + 4)
      NI2 = NI1 + 1
      DO 8 J = NI2,N
8     C(NI1+4) = C(NI1+4) - C(J+4)*A(NI1*NP1 + J - NP3)
      C(NI1+4) = C(NI1+4)/A(NI1*NP1 + NI1 - NP3)
16     DETA = DETA*A(I*NP1 + I - NP3)
      PRINT 101, DETA
      C(1) = 1.0
      C(2) = A(1) - 1.0
      C(3) = C(2)
      C(4) = 0.0
      RETURN
      END
```

APPENDIX A – Continued

Subroutine SPIT(A,B)

Purpose: To print array A

Flow chart:



Description: Subroutine SPIT prints a heading which includes the number of rows and columns of array A and then prints the array row after row in a straightforward manner. A vector format is used that was described earlier

Program variables: A array to be printed

II number of rows

JJ number of columns

Subroutine listing:

```
SUBROUTINE SPIT(A, B)
C  VECTOR FORMAT
      DIMENSION A(1)
505  FORMAT(5E12.4)
321  FORMAT(8H MATRIX ,A5,4H HAS ,I4,10H ROWS AND ,I4,8H COLUMNS)
      II = A(1) + .01
      JJ = A(2) + .01
      PRINT 321, B, II, JJ
      L = 4 - JJ
      IF(II)3,3,2
2   DO 1 I = 1,II
      L = L + JJ
1   PRINT 505,(A(L+J), J = 1,JJ)
3   RETURN
      END
```

APPENDIX A – Continued

Subroutine TRICK(A,B,C)

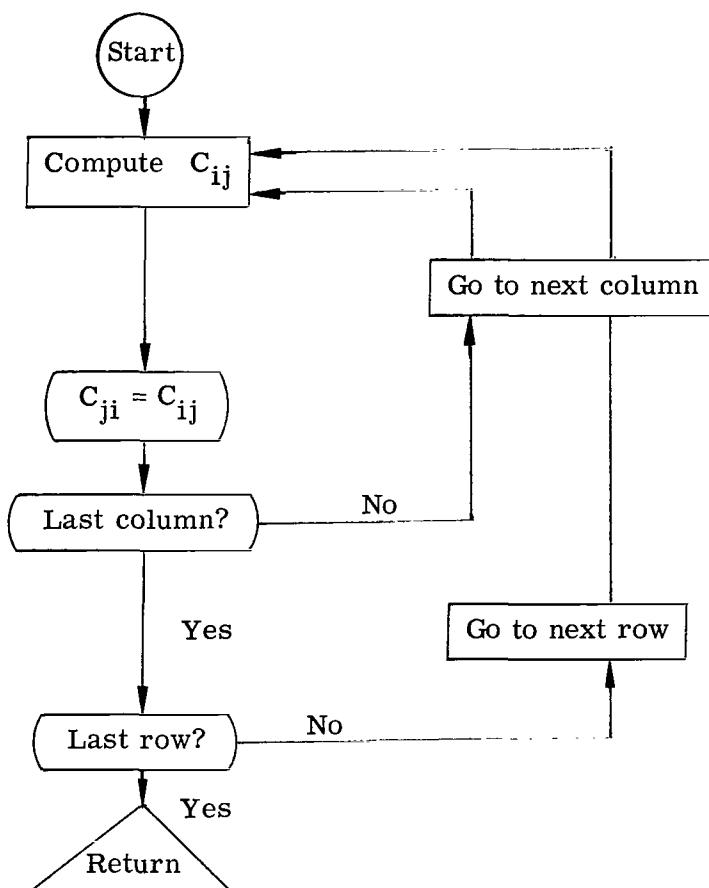
Purpose: To form a matrix of coefficients of a system of simultaneous linear algebraic equations

Formula: $C_{n+1} = C_n + A^T B' A$

where

$$B' = \begin{bmatrix} B_1 & & 0 \\ & B_2 & \\ & & B_3 \\ 0 & & B_{II} \end{bmatrix}$$

Flow chart:



APPENDIX A – Concluded

Description: Subroutine TRICK is designed for use in evaluating a matrix which is involved typically when a system of linear simultaneous algebraic equations is formed. A vector format which was described earlier is used for all the arrays.

<u>Program variables:</u>	A	matrix to be "squared"
	B	weighting matrix
	C	simulation matrix
	II	number of rows in A and C
	JJ	number of columns in A and B
	I,J,IJ	general indices

Subroutine listing:

```
SUBROUTINE TRICK(A, B, C)
C   VECTOR FORMAT
      DIMENSION A(1), B(1), C(1)
      IB = B(1) + .01
      JB = B(2) + .01
      II = A(1) + .01
      JJ = A(2) + .01
      IF((IB-1)*(JB-1))3,4,3
4  CONTINUE
      DO 1 J = 1,II
      DO 1 I = J,II
      IJ = (I-1)*II + J + 4
      DO 2 K = 1,JJ
2  C(IJ) = C(IJ) + A(I*JJ+K-JJ+4)*A(J*JJ+K-JJ+4)*B(K+4)
1  C(J*II + I - II + 4) = C(IJ)
      RETURN
3  CONTINUE
      DO 11 J = 1,II
      DO 11 I = J,II
      IJ = (I-1)*II + J + 4
      DO 12 K = 1,JJ
      IK = (I-1)*JJ + K + 4
      DO 12 L = 1,JJ
      KL = (K-1)*JJ + L + 4
      JL = (J-1)*JJ + L + 4
12  C(IJ) = C(IJ) + A(IK)*B(KL)*A(JL)
11  C(J*II + I - II + 4) = C(IJ)
      RETURN
      END
```

APPENDIX B

INPUT AND OUTPUT LISTINGS

The input and output listings of this appendix are provided as an aid to the user. The user should first compute the example discussed in the text; then the input can be changed to suit the particular needs of the user.

Input Listing

	200	10				
.05		.01				
A	4	4				
	-1.0					
1.0						
B	4	3				
D1	1	7				
1.0	1.0	1.0				
F	3	4				
O.	0.	0.				
O.	0.	0.				
	-1.0					
G	3	3				
O.	0.	0.				
O.	0.	0.				
O.	0.	0.				
SZ	1	7				
1.0	1.0	1.0				
SU	1	3				
1.0	1.0	1.0				
LP	1	1	1	1	99999.	1.0
LP		5	1	1		1.0
LR	1	1	1	2	99999.	1.0
LR		5	1	2		1.0
LB	1	1	1	3	99999.	1.0
LB		5	1	3		1.0
LDA	1	2	1	1	99999.	1.0
LDA		6	1	1		1.0
LDR	1	2	1	2	99999.	1.0
LDR		6	1	2		1.0
LO	1	2	1	3	99999.	1.0
LO		6	1	3		1.0
NP	1	1	2	1	99999.	1.0
NP		5	2	1		1.0
NR	1	1	2	2	99999.	1.0
NR		5	2	2		1.0
NB	1	1	2	3	99999.	1.0
NB		5	2	3		1.0

APPENDIX B – Continued

NDA	1	2	2	1	99999.	1.0
NDA		6	2	1		1.0
NDR	1	2	2	2	99999.	1.0
NDR		6	2	2		1.0
NO	1	2	2	3	99999.	1.0
NO		6	2	3		1.0
YP	1	1	3	1	99999.	1.0
YP		5	3	1		1.0
YB	1	1	3	3	99999.	1.0
YB		5	3	3		1.0
YO	1	2	3	3	99999.	1.0
YO		6	3	3		1.0
END	-1					
0.	0.	1.0000E+00	8.9372E-05	-4.1952E-05	2.2927E-05	
2.2944E-06	1.6415E-03	-8.0941E-04	4.7933E-04	5.0000E-02		
0.	0.	1.0000E+00	1.6384E-04	-8.0879E-05	4.7857E-05	
8.6888E-06	1.3345E-03	-7.4711E-04	5.1709E-04	1.0000E-01		
0.	0.	1.0000E+00	2.2256E-04	-1.1661E-04	7.4556E-05	
1.8416E-05	1.0118E-03	-6.8167E-04	5.5005E-04	1.5000E-01		
0.	0.	1.0000E+00	2.6481E-04	-1.4900E-04	1.0278E-04	
3.0670E-05	6.7648E-04	-6.1363E-04	5.7803E-04	2.0000E-01		
0.	0.	1.0000E+00	2.9005E-04	-1.7794E-04	1.3227E-04	
4.4614E-05	3.3153E-04	-5.4357E-04	6.0087E-04	2.5000E-01		
0.	0.	1.0000E+00	2.9786E-04	-2.0334E-04	1.6278E-04	
5.9385E-05	-1.9951E-05	-4.7208E-04	6.1847E-04	3.0000E-01		
0.	0.	1.0000E+00	2.8800E-04	-2.2513E-04	1.9403E-04	
7.4105E-05	-3.7489E-04	-3.9974E-04	6.3076E-04	3.5000E-01		
0.	0.	1.0000E+00	2.6036E-04	-2.4331E-04	2.2577E-04	
8.7888E-05	-7.3023E-04	-3.2714E-04	6.3773E-04	4.0000E-01		
0.	0.	1.0000E+00	2.1502E-04	-2.5785E-04	2.5772E-04	
9.9846E-05	-1.0829E-03	-2.5484E-04	6.3940E-04	4.5000E-01		
0.	0.	1.0000E+00	1.5216E-04	-2.6880E-04	2.8962E-04	
1.0910E-04	-1.4301E-03	-1.8342E-04	6.3581E-04	5.0000E-01		
0.	0.	1.0000E+00	7.2150E-05	-2.7622E-04	3.2121E-04	
1.1478E-04	-1.7688E-03	-1.1342E-04	6.2708E-04	5.5000E-01		
0.	0.	1.0000E+00	-2.4530E-05	-2.8018E-04	3.5224E-04	
1.1604E-04	-2.0963E-03	-4.5389E-05	6.1334E-04	6.0000E-01		
0.	0.	1.0000E+00	-1.3725E-04	-2.8080E-04	3.8247E-04	
1.1206E-04	-2.4100E-03	2.0174E-05	5.9477E-04	6.5000E-01		
0.	0.	1.0000E+00	-2.6526E-04	-2.7821E-04	4.1164E-04	
1.0206E-04	-2.7074E-03	8.2782E-05	5.7158E-04	7.0000E-01		
0.	0.	1.0000E+00	-4.0768E-04	-2.7258E-04	4.3955E-04	
8.5290E-05	-2.9861E-03	1.4198E-04	5.4401E-04	7.5000E-01		
0.	0.	1.0000E+00	-5.6353E-04	-2.6408E-04	4.6598E-04	
6.1064E-05	-3.2442E-03	1.9735E-04	5.1234E-04	8.0000E-01		
0.	5.0000E-01	1.0000E+00	5.0921E-02	-4.7040E-02	1.4466E-03	
9.9991E-04	2.0536E+00	-1.8679E+00	5.1327E-02	8.5000E-01		
0.	1.0000E+00	1.0000E+00	2.0320E-01	-1.8650E-01	7.4774E-03	
7.0519E-03	4.0194E+00	-3.7056E+00	2.0201E-01	9.0000E-01		
4.9000E+00	1.0000E+00	1.0000E+00	1.3997E+00	-5.0905E-01	2.6685E-02	
4.0948E-02	4.3730E+01	-9.1812E+00	6.2049E-01	9.5000E-01		
9.8000E+00	1.0000E+00	1.0000E+00	4.5598E+00	-1.1024E+00	7.7664E-02	
1.8395E-01	8.2525E+01	-1.4529E+01	1.4713E+00	1.0000E+00		
9.5500E+00	1.0000E+00	1.0000E+00	8.5798E+00	-1.8129E+00	1.7739E-01	
5.1324E-01	7.8187E+01	-1.3875E+01	2.5101E+00	1.0500E+00		
9.3000E+00	1.0000E+00	1.0000E+00	1.2370E+01	-2.4881E+00	3.2771E-01	
1.0379E+00	7.3338E+01	-1.3118E+01	3.4939E+00	1.1000E+00		
2.5000E+00	1.0000E+00	1.0000E+00	1.4568E+01	-2.9374E+00	5.2013E-01	
1.7207E+00	1.4618E+01	-4.8520E+00	4.1188E+00	1.1500E+00		
-4.3000E+00	1.0000E+00	1.0000E+00	1.3844E+01	-2.9745E+00	7.2736E-01	

APPENDIX B – Continued

2.4401E+00	-4.3445E+01	3.3565E+00	4.0874E+00	i.2000E+00	
-4.3333E+00	1.0000E+00	1.0000E+00	1.1624E+01	-2.7959E+00	9.2251E-01
3.0772E+00	-4.5324E+01	3.7779E+00	3.7134E+00	1.2500E+00	
-4.3667E+00	1.0000E+00	1.0000E+00	9.3161E+00	-2.5974E+00	1.0982E+00
3.6011E+00	-4.6956E+01	4.1528E+00	3.3093E+00	1.3000E+00	
-4.4000E+00	1.0000E+00	1.0000E+00	6.9329E+00	-2.3815E+00	1.2530E+00
4.0076E+00	-4.8328E+01	4.4786E+00	2.8788E+00	1.3500E+00	
-4.4333E+00	1.0000E+00	1.0000E+00	4.4878E+00	-2.1504E+00	1.3857E+00
4.2933E+00	-4.9431E+01	4.7533E+00	2.4256E+00	1.4000E+00	
-4.4667E+00	1.0000E+00	1.0000E+00	1.9945E+00	-1.9070E+00	1.4952E+00
4.4555E+00	-5.0255E+01	4.9751E+00	1.9535E+00	1.4500E+00	
-4.5000E+00	1.0000E+00	1.0000E+00	-5.3294E-01	-1.6538E+00	1.5808E+00
4.4922E+00	-5.0796E+01	5.1427E+00	1.4665E+00	1.5000E+00	
-2.2000E+00	1.0000E+00	1.0000E+00	-2.6036E+00	-1.4598E+00	1.6437E+00
4.4108E+00	-3.2028E+01	2.6141E+00	1.0744E+00	1.5500E+00	
1.0000E-01	1.0000E+00	1.0000E+00	-3.7379E+00	-1.3924E+00	1.6920E+00
4.2494E+00	-1.3358E+01	8.1551E-02	8.8490E-01	1.6000E+00	
1.0000E-01	8.0833E-01	1.0000E+00	-4.4332E+00	-1.3681E+00	1.7337E+00
4.0453E+00	-1.4440E+01	8.8655E-01	7.7370E-01	1.6500E+00	
1.0000E-01	6.1667E-01	1.0000E+00	-5.1803E+00	-1.3042E+00	1.7686E+00
3.8051E+00	-1.5428E+01	1.6687E+00	6.1776E-01	1.7000E+00	
1.0000E-01	4.2500E-01	1.0000E+00	-5.9740E+00	-1.2017E+00	1.7946E+00
3.5264E+00	-1.6301E+01	2.4236E+00	4.1926E-01	1.7500E+00	
1.0000E-01	2.3333E-01	1.0000E+00	-6.8080E+00	-1.0624E+00	1.8098E+00
3.2069E+00	-1.7035E+01	3.1470E+00	1.8068E-01	1.8000E+00	
1.0000E-01	4.1667E-02	1.0000E+00	-7.6749E+00	-8.8766E-01	1.8120E+00
2.8450E+00	-1.7612E+01	3.8350E+00	-9.5236E-02	1.8500E+00	
1.0000E-01	-1.5000E-01	1.0000E+00	-8.5662E+00	-6.7951E-01	1.7996E+00
2.4390E+00	-1.8012E+01	4.4843E+00	-4.0548E-01	1.9000E+00	
1.0000E-01	-3.4167E-01	1.0000E+00	-9.4728E+00	-4.3995E-01	1.7709E+00
1.9880E+00	-1.8220E+01	5.0916E+00	-7.4683E-01	1.9500E+00	
1.0000E-01	-5.3333E-01	1.0000E+00	-1.0385E+01	-1.7112E-01	1.7244E+00
1.4915E+00	-1.8223E+01	5.6542E+00	-1.1159E+00	2.0000E+00	
1.0000E-01	-7.2500E-01	1.0000E+00	-1.1291E+01	1.2468E-01	1.6588E+00
9.4952E-01	-1.8008E+01	6.1699E+00	-1.5090E+00	2.0500E+00	
1.0000E-01	-9.1667E-01	1.0000E+00	-1.2182E+01	4.4504E-01	1.5730E+00
3.6255E-01	-1.7566E+01	6.6366E+00	-1.9226E+00	2.1000E+00	
1.0000E-01	-1.1083E+00	1.0000E+00	-1.3044E+01	7.8748E-01	1.4662E+00
-2.6828E-01	-1.6889E+01	7.0530E+00	-2.3527E+00	2.1500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.3867E+01	1.1495E+00	1.3375E+00
-9.4128E-01	-1.5973E+01	7.4180E+00	-2.7955E+00	2.2000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.4618E+01	1.5105E+00	1.1868E+00
-1.6538E+00	-1.4027E+01	7.0149E+00	-3.2276E+00	2.2500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.5266E+01	1.8503E+00	1.0153E+00
-2.4013E+00	-1.1873E+01	6.5712E+00	-3.6262E+00	2.3000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.5802E+01	2.1670E+00	8.2474E-01
-3.1785E+00	-9.5344E+00	6.0912E+00	-3.9887E+00	2.3500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.6217E+01	2.4589E+00	6.1704E-01
-3.9795E+00	-7.0347E+00	5.5792E+00	-4.3128E+00	2.4000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.6503E+01	2.7245E+00	3.9413E-01
-4.7980E+00	-4.3986E+00	5.0399E+00	-4.5965E+00	2.4500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.6655E+01	2.9625E+00	1.5809E-01
-5.6275E+00	-1.6513E+00	4.4780E+00	-4.8382E+00	2.5000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.6667E+01	3.1720E+00	-8.8965E-02
-6.4612E+00	1.1814E+00	3.8983E+00	-5.0366E+00	2.5500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.6535E+01	3.3521E+00	-3.4484E-01
-7.2918E+00	4.0736E+00	3.3058E+00	-5.1908E+00	2.6000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.6259E+01	3.5024E+00	-6.0730E-01
-8.1123E+00	6.9992E+00	2.7053E+00	-5.3002E+00	2.6500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.5835E+01	3.6226E+00	-8.7411E-01
-8.9152E+00	9.9323E+00	2.1019E+00	-5.3646E+00	2.7000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.5266E+01	3.7126E+00	-1.1430E+00

APPENDIX B – Continued

-9.6934E+00	1.2847E+01	1.5004E+00	-5.3840E+00	2.7500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.4551E+01	3.7727E+00	-1.4118E+00
-1.0439E+01	1.5719E+01	9.0566E-01	-5.3590E+00	2.8000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.3695E+01	3.8034E+00	-1.6782E+00
-1.1146E+01	1.8524E+01	3.2229E-01	-5.2904E+00	2.8500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.2701E+01	3.8052E+00	-1.9401E+00
-1.1807E+01	2.1237E+01	-2.4515E-01	-5.1794E+00	2.9000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.1573E+01	3.7792E+00	-2.1954E+00
-1.2414E+01	2.3837E+01	-7.9231E-01	-5.0272E+00	2.9500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-1.0319E+01	3.7264E+00	-2.4422E+00
-1.2962E+01	2.6303E+01	-1.3151E+00	-4.8359E+00	3.0000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-8.9454E+00	3.6482E+00	-2.6784E+00
-1.3444E+01	2.8615E+01	-1.8096E+00	-4.6072E+00	3.0500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-7.4605E+00	3.5460E+00	-2.9023E+00
-1.3855E+01	3.0754E+01	-2.2723E+00	-4.3437E+00	3.1000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-5.8732E+00	3.4215E+00	-3.1122E+00
-1.4188E+01	3.2705E+01	-2.6998E+00	-4.0477E+00	3.1500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-4.1934E+00	3.2766E+00	-3.3066E+00
-1.4440E+01	3.4451E+01	-3.0892E+00	-3.7220E+00	3.2000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-2.4317E+00	3.1133E+00	-3.4840E+00
-1.4606E+01	3.5981E+01	-3.4379E+00	-3.3696E+00	3.2500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	-5.9913E-01	2.9336E+00	-3.6432E+00
-1.4682E+01	3.7282E+01	-3.7435E+00	-2.9937E+00	3.3000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	1.2926E+00	2.7397E+00	-3.7830E+00
-1.4665E+01	3.8346E+01	-4.0042E+00	-2.5974E+00	3.3500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.2314E+00	2.5339E+00	-3.9026E+00
-1.4552E+01	3.9165E+01	-4.2183E+00	-2.1841E+00	3.4000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	5.2049E+00	2.3186E+00	-4.0012E+00
-1.4341E+01	3.9735E+01	-4.3849E+00	-1.7574E+00	3.4500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	7.2006E+00	2.0962E+00	-4.0782E+00
-1.4031E+01	4.0052E+01	-4.5030E+00	-1.3206E+00	3.5000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	9.2059E+00	1.8692E+00	-4.1332E+00
-1.3621E+01	4.0115E+01	-4.5724E+00	-8.7740E-01	3.5500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	1.1208E+01	1.6398E+00	-4.1659E+00
-1.3111E+01	3.9925E+01	-4.5931E+00	-4.3131E-01	3.6000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	1.3194E+01	1.4106E+00	-4.1763E+00
-1.2501E+01	3.9485E+01	-4.5654E+00	1.4138E-02	3.6500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	1.5152E+01	1.1841E+00	-4.1645E+00
-1.1792E+01	3.8801E+01	-4.4902E+00	4.5547E-01	3.7000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	1.7070E+01	9.6240E-01	-4.1309E+00
-1.0986E+01	3.7878E+01	-4.3685E+00	8.8929E-01	3.7500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	1.8936E+01	7.4796E-01	-4.0758E+00
-1.0086E+01	3.6725E+01	-4.2019E+00	1.3123E+00	3.8000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	2.0739E+01	5.4293E-01	-3.9999E+00
-9.0935E+00	3.5353E+01	-3.9922E+00	1.7213E+00	3.8500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	2.2468E+01	3.4943E-01	-3.9039E+00
-8.0130E+00	3.3773E+01	-3.7414E+00	2.1134E+00	3.9000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	2.4113E+01	1.6943E-01	-3.7889E+00
-6.8481E+00	3.2000E+01	-3.4521E+00	2.4856E+00	3.9500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	2.5665E+01	4.8128E-03	-3.6557E+00
-5.6033E+00	3.0047E+01	-3.1270E+00	2.8353E+00	4.0000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	2.7115E+01	-1.4271E-01	-3.5058E+00
-4.2833E+00	2.7930E+01	-2.7689E+00	3.1601E+00	4.0500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	2.8456E+01	-2.7158E-01	-3.3402E+00
-2.8936E+00	2.5667E+01	-2.3810E+00	3.4577E+00	4.1000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	2.9680E+01	-3.8038E-01	-3.1605E+00
-1.4397E+00	2.3276E+01	-1.9668E+00	3.7262E+00	4.1500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.0781E+01	-4.6788E-01	-2.9681E+00
7.2358E-02	2.0776E+01	-1.5297E+00	3.9638E+00	4.2000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.1756E+01	-5.3302E-01	-2.7646E+00

APPENDIX B – Continued

1.6363E+00	1.8185E+01	-1.0733E+00	4.1692E+00	4.2500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.2599E+01	-5.7495E-01	-2.5517E+00
3.2458E+00	1.5525E+01	-6.0151E-01	4.3410E+00	4.3000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.3307E+01	-5.9298E-01	-2.3311E+00
4.8940E+00	1.2814E+01	-1.1806E-01	4.4784E+00	4.3500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.3880E+01	-5.8663E-01	-2.1045E+00
6.5742E+00	1.0074E+01	3.7315E-01	4.5807E+00	4.4000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.4315E+01	-5.5560E-01	-1.8736E+00
8.2797E+00	7.3254E+00	8.6823E-01	4.6476E+00	4.4500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.4613E+01	-4.9981E-01	-1.6403E+00
1.0003E+01	4.5875E+00	1.3633E+00	4.6790E+00	4.5000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.4774E+01	-4.1933E-01	-1.4063E+00
1.1739E+01	1.8806E+00	1.8546E+00	4.6752E+00	4.5500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.4801E+01	-3.1447E-01	-1.1734E+00
1.3479E+01	-7.7582E-01	2.3383E+00	4.6365E+00	4.6000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.4698E+01	-1.8569E-01	-9.4323E-01
1.5217E+01	-3.3630E+00	2.8108E+00	4.5637E+00	4.6500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.4467E+01	-3.3644E-02	-7.1755E-01
1.6946E+01	-5.8629E+00	3.2685E+00	4.4579E+00	4.7000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.4113E+01	1.4086E-01	-4.9797E-01
1.8661E+01	-8.2584E+00	3.7083E+00	4.3202E+00	4.7500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.3643E+01	3.3683E-01	-2.8604E-01
2.0356E+01	-1.0533E+01	4.1268E+00	4.1521E+00	4.8000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.3062E+01	5.5313E-01	-8.3234E-02
2.2024E+01	-1.2673E+01	4.5211E+00	3.9553E+00	4.8500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.2378E+01	7.8849E-01	1.0905E-01
2.3660E+01	-1.4663E+01	4.8885E+00	3.7317E+00	4.9000E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.1598E+01	1.0415E+00	2.8953E-01
2.5260E+01	-1.6491E+01	5.2265E+00	3.4834E+00	4.9500E+00	
1.0000E-01	-1.3000E+00	1.0000E+00	3.0732E+01	1.3106E+00	4.5702E-01
2.6818E+01	-1.8146E+01	5.5328E+00	3.2126E+00	5.0000E+00	
-2.3500E+00	-1.3000E+00	1.0000E+00	2.9286E+01	1.6637E+00	6.0836E-01
2.8322E+01	-3.9592E+01	8.5789E+00	2.8105E+00	5.0500E+00	
-4.8000E+00	-1.3000E+00	1.0000E+00	2.6783E+01	2.1670E+00	7.3364E-01
2.9727E+01	-6.0442E+01	1.1536E+01	2.1709E+00	5.1000E+00	
-7.2500E+00	-1.3000E+00	1.0000E+00	2.3255E+01	2.8153E+00	8.2115E-01
3.0981E+01	-8.0572E+01	1.4379E+01	1.3010E+00	5.1500E+00	
-9.7000E+00	-1.3000E+00	1.0000E+00	1.8740E+01	3.6025E+00	8.5959E-01
3.2034E+01	-9.9863E+01	1.7086E+01	2.1011E-01	5.2000E+00	
-9.5000E+00	-9.0000E-01	1.0000E+00	1.3868E+01	4.4086E+00	8.4121E-01
3.2849E+01	-9.4956E+01	1.5140E+01	-9.3039E-01	5.2500E+00	
-9.3000E+00	-5.0000E-01	1.0000E+00	9.2538E+00	5.1149E+00	7.6876E-01
3.3426E+01	-8.9510E+01	1.3098E+01	-1.9513E+00	5.3000E+00	
-7.9000E+00	-5.0000E-01	1.0000E+00	5.1280E+00	5.7205E+00	6.4863E-01
3.3783E+01	-7.5464E+01	1.1110E+01	-2.8332E+00	5.3500E+00	
-6.5000E+00	-5.0000E-01	1.0000E+00	1.7113E+00	6.2248E+00	4.8833E-01
3.3952E+01	-6.1161E+01	9.0563E+00	-3.5574E+00	5.4000E+00	
-5.1000E+00	-5.0000E-01	1.0000E+00	-9.8575E-01	6.6253E+00	2.9585E-01
3.3967E+01	-4.6695E+01	6.9558E+00	-4.1199E+00	5.4500E+00	
-3.7000E+00	-5.0000E-01	1.0000E+00	-2.9574E+00	6.9200E+00	7.9348E-02
3.3866E+01	-3.2160E+01	4.8261E+00	-4.5179E+00	5.5000E+00	
-2.3000E+00	-5.0000E-01	1.0000E+00	-4.2025E+00	7.1078E+00	-1.5292E-01
3.3685E+01	-1.7652E+01	2.6853E+00	-4.7503E+00	5.5500E+00	
-9.0000E-01	-5.0000E-01	1.0000E+00	-4.7249E+00	7.1887E+00	-3.9265E-01
3.3459E+01	-3.2624E+00	5.5161E-01	-4.8168E+00	5.6000E+00	
5.0000E-01	-5.0000E-01	1.0000E+00	-4.5326E+00	7.1634E+00	-6.3158E-01
3.3225E+01	1.0915E+01	-1.5574E+00	-4.7187E+00	5.6500E+00	
1.9000E+00	-5.0000E-01	1.0000E+00	-3.6387E+00	7.0337E+00	-8.6155E-01
3.3019E+01	2.4792E+01	-3.6242E+00	-4.4587E+00	5.7000E+00	
3.3000E+00	-5.0000E-01	1.0000E+00	-2.0603E+00	6.8020E+00	-1.0745E+00
3.2874E+01	3.8281E+01	-5.6322E+00	-4.0403E+00	5.7500E+00	
4.7000E+00	-5.0000E-01	1.0000E+00	1.8122E-01	6.4717E+00	-1.2628E+00
3.2825E+01	5.1300E+01	-7.5649E+00	-3.4686E+00	5.8000E+00	

APPENDIX B – Continued

6.1000E+00	-5.0000E-01	1.0000E+00	3.0602E+00	6.0470E+00	-1.4187E+00
3.2904E+01	6.3770E+01	-9.4072E+00	-2.7496E+00	5.8500E+00	
7.5000E+00	-5.0000E-01	1.0000E+00	6.5475E+00	5.5328E+00	-1.5351E+00
3.3142E+01	7.5618E+01	-1.1144E+01	-1.8904E+00	5.9000E+00	
8.9000E+00	-5.0000E-01	1.0000E+00	1.0610E+01	4.9346E+00	-1.6053E+00
3.3570E+01	8.6775E+01	-1.2763E+01	-8.9924E-01	5.9500E+00	
1.0300E+01	-5.0000E-01	1.0000E+00	1.5212E+01	4.2588E+00	-1.6227E+00
3.4214E+01	9.7179E+01	-1.4250E+01	2.1490E-01	6.0000E+00	
1.0400E+01	-5.0000E-01	1.0000E+00	2.0049E+01	3.5489E+00	-1.5827E+00
3.5095E+01	9.6176E+01	-1.4123E+01	1.3832E+00	6.0500E+00	
1.0500E+01	-5.0000E-01	1.0000E+00	2.4819E+01	2.8485E+00	-1.4846E+00
3.6218E+01	9.4529E+01	-1.3872E+01	2.5371E+00	6.1000E+00	
1.0600E+01	-5.0000E-01	1.0000E+00	2.9491E+01	2.1637E+00	-1.3294E+00
3.7576E+01	9.2258E+01	-1.3501E+01	3.6680E+00	6.1500E+00	
1.0700E+01	-5.0000E-01	1.0000E+00	3.4035E+01	1.5004E+00	-1.1183E+00
3.9165E+01	8.9388E+01	-1.3013E+01	4.7673E+00	6.2000E+00	
9.5000E+00	-5.0000E-01	1.0000E+00	3.8155E+01	9.0115E-01	-8.5440E-01
4.0972E+01	7.5349E+01	-1.0942E+01	5.7679E+00	6.2500E+00	
8.3000E+00	-5.0000E-01	1.0000E+00	4.1565E+01	4.0744E-01	-5.4451E-01
4.2967E+01	6.0987E+01	-8.7943E+00	6.6045E+00	6.3000E+00	
7.1000E+00	-5.0000E-01	1.0000E+00	4.4250E+01	2.2651E-02	-1.9700E-01
4.5115E+01	4.6402E+01	-6.5886E+00	7.2722E+00	6.3500E+00	
5.9000E+00	-5.0000E-01	1.0000E+00	4.6203E+01	-2.5080E-01	1.7961E-01
4.7379E+01	3.1692E+01	-4.3440E+00	7.7677E+00	6.4000E+00	
4.7000E+00	-5.0000E-01	1.0000E+00	4.7419E+01	-4.1144E-01	5.7664E-01
4.9722E+01	1.6959E+01	-2.0794E+00	8.0891E+00	6.4500E+00	
3.5000E+00	-5.0000E-01	1.0000E+00	4.7900E+01	-4.5876E-01	9.8538E-01
5.2108E+01	2.3003E+00	1.8606E-01	8.2358E+00	6.5000E+00	
2.3000E+00	-5.0000E-01	1.0000E+00	4.7652E+01	-3.9317E-01	1.3971E+00
5.4499E+01	-1.2185E+01	2.4335E+00	8.2086E+00	6.5500E+00	
1.1000E+00	-5.0000E-01	1.0000E+00	4.6687E+01	-2.1604E-01	1.8032E+00
5.6860E+01	-2.6403E+01	4.6445E+00	8.0097E+00	6.6000E+00	
-1.0000E-01	-5.0000E-01	1.0000E+00	4.5018E+01	7.0346E-02	2.1950E+00
5.9155E+01	-4.0259E+01	6.8012E+00	7.6425E+00	6.6500E+00	
-1.3000E+00	-5.0000E-01	1.0000E+00	4.2668E+01	4.6284E-01	2.5645E+00
6.1349E+01	-5.3666E+01	8.8860E+00	7.1117E+00	6.7000E+00	
-2.5000E+00	-5.0000E-01	1.0000E+00	3.9661E+01	9.5745E-01	2.9034E+00
6.3410E+01	-6.6539E+01	1.0883E+01	6.4233E+00	6.7500E+00	
-3.7000E+00	-5.0000E-01	1.0000E+00	3.6025E+01	1.5493E+00	3.2040E+00
6.5304E+01	-7.8798E+01	1.2775E+01	5.5844E+00	6.8000E+00	
-3.7000E+00	9.0000E-01	1.0000E+00	3.2183E+01	2.0679E+00	3.4629E+00
6.7009E+01	-7.4825E+01	7.9600E+00	4.7999E+00	6.8500E+00	
-3.7000E+00	2.3000E+00	1.0000E+00	2.8547E+01	2.3451E+00	3.6889E+00
6.8526E+01	-7.0550E+01	3.1239E+00	4.2695E+00	6.9000E+00	
-3.7000E+00	3.7000E+00	1.0000E+00	2.5130E+01	2.3805E+00	3.8946E+00
6.9868E+01	-6.6119E+01	-1.7055E+00	3.9914E+00	6.9500E+00	
-3.7000E+00	5.1000E+00	1.0000E+00	2.1935E+01	2.1752E+00	4.0928E+00
7.1044E+01	-6.1677E+01	-6.5011E+00	3.9620E+00	7.0000E+00	
-3.7000E+00	5.0900E+00	1.0000E+00	1.8815E+01	1.8635E+00	4.2928E+00
7.2063E+01	-6.3164E+01	-5.9687E+00	4.0321E+00	7.0500E+00	
-3.7000E+00	5.0800E+00	1.0000E+00	1.5619E+01	1.5784E+00	4.4952E+00
7.2924E+01	-6.4655E+01	-5.4368E+00	4.0562E+00	7.1000E+00	
-3.7000E+00	5.0700E+00	1.0000E+00	1.2349E+01	1.3197E+00	4.6977E+00
7.3623E+01	-6.6125E+01	-4.9104E+00	4.0348E+00	7.1500E+00	
-3.7000E+00	5.0600E+00	1.0000E+00	9.0073E+00	1.0872E+00	4.8979E+00
7.4158E+01	-6.7549E+01	-4.3942E+00	3.9691E+00	7.2000E+00	
-3.7000E+00	5.0500E+00	1.0000E+00	5.5957E+00	8.8004E-01	5.0938E+00
7.4523E+01	-6.8902E+01	-3.8929E+00	3.8601E+00	7.2500E+00	
-3.7000E+00	5.0400E+00	1.0000E+00	2.1187E+00	6.9754E-01	5.2832E+00
7.4716E+01	-7.0161E+01	-3.4109E+00	3.7096E+00	7.3000E+00	
-3.7000E+00	5.0300E+00	1.0000E+00	-1.4185E+00	5.3856E-01	5.4641E+00
7.4734E+01	-7.1306E+01	-2.9524E+00	3.5194E+00	7.3500E+00	
-3.7000E+00	5.0200E+00	1.0000E+00	-5.0096E+00	4.0184E-01	5.6346E+00

APPENDIX B – Continued

7.4573E+01	-7.2315E+01	-2.5213E+00	3.2917E+00	7.4000E+00		
-3.7000E+00	5.0100E+00	1.0000E+00	-8.6474E+00	2.8591E-01	5.7927E+00	
7.4232E+01	-7.3170E+01	-2.1212E+00	3.0290E+00	7.4500E+00		
-3.7000E+00	5.0000E+00	1.0000E+00	-1.2324E+01	1.8915E-01	5.9369E+00	
7.3708E+01	-7.3854E+01	-1.7554E+00	2.7338E+00	7.5000E+00		
-3.7000E+00	4.9900E+00	1.0000E+00	-1.6030E+01	1.0975E-01	6.0656E+00	
7.2999E+01	-7.4353E+01	-1.4269E+00	2.4092E+00	7.5500E+00		
-3.7000E+00	4.9800E+00	1.0000E+00	-1.9756E+01	4.5794E-02	6.1774E+00	
7.2105E+01	-7.4652E+01	-1.1383E+00	2.0581E+00	7.6000E+00		
-3.7000E+00	4.9700E+00	1.0000E+00	-2.3491E+01	-4.7846E-03	6.2710E+00	
7.1023E+01	-7.4740E+01	-8.9197E-01	1.6838E+00	7.6500E+00		
-3.7000E+00	4.9600E+00	1.0000E+00	-2.7226E+01	-4.4138E-02	6.3454E+00	
6.9755E+01	-7.4610E+01	-6.8964E-01	1.2897E+00	7.7000E+00		
-3.7000E+00	4.9500E+00	1.0000E+00	-3.0949E+01	-7.4509E-02	6.3997E+00	
6.8301E+01	-7.4252E+01	-5.3285E-01	8.7925E-01	7.7500E+00		
-3.7000E+00	4.9400E+00	1.0000E+00	-3.4647E+01	-9.8201E-02	6.4331E+00	
6.6661E+01	-7.3663E+01	-4.2267E-01	4.5601E-01	7.8000E+00		
-3.7000E+00	4.9300E+00	1.0000E+00	-3.8311E+01	-1.1756E-01	6.4452E+00	
6.4837E+01	-7.2839E+01	-3.5980E-01	2.3589E-02	7.8500E+00		
-3.7000E+00	4.9200E+00	1.0000E+00	-4.1927E+01	-1.3497E-01	6.4354E+00	
6.2831E+01	-7.1780E+01	-3.4452E-01	-4.1438E-01	7.9000E+00		
-3.7000E+00	4.9100E+00	1.0000E+00	-4.5485E+01	-1.5281E-01	6.4037E+00	
6.0645E+01	-7.0486E+01	-3.7673E-01	-8.5428E-01	7.9500E+00		
-3.7000E+00	4.9000E+00	1.0000E+00	-4.8972E+01	-1.7343E-01	6.3500E+00	
5.8283E+01	-6.8961E+01	-4.5595E-01	-1.2925E+00	8.0000E+00		
-3.7000E+00	3.4000E+00	1.0000E+00	-5.2531E+01	-5.9748E-02	6.2717E+00	
5.5746E+01	-7.3341E+01	4.9857E+00	-1.8771E+00	8.0500E+00		
-3.7000E+00	1.9000E+00	1.0000E+00	-5.6298E+01	3.2276E-01	6.1568E+00	
5.3026E+01	-7.7231E+01	1.0292E+01	-2.7507E+00	8.1000E+00		
-3.7000E+00	4.0000E-01	1.0000E+00	-6.0244E+01	9.6659E-01	5.9914E+00	
5.0113E+01	-8.0479E+01	1.5434E+01	-3.8997E+00	8.1500E+00		
-3.7000E+00	-1.1000E+00	1.0000E+00	-6.4332E+01	1.8628E+00	5.7619E+00	
4.6999E+01	-8.2941E+01	2.0384E+01	-5.3088E+00	8.2000E+00		
-3.4429E+00	-1.0357E+00	1.0000E+00	-6.8308E+01	2.8476E+00	5.4590E+00	
4.3681E+01	-7.5956E+01	1.8982E+01	-6.7898E+00	8.2500E+00		
-3.1857E+00	-9.7143E-01	1.0000E+00	-7.1916E+01	3.7587E+00	5.0848E+00	
4.0174E+01	-6.8263E+01	1.7441E+01	-8.1593E+00	8.3000E+00		
-2.9286E+00	-9.0714E-01	1.0000E+00	-7.5124E+01	4.5896E+00	4.6452E+00	
3.6497E+01	-5.9939E+01	1.5775E+01	-9.4081E+00	8.3500E+00		
-2.6714E+00	-8.4286E-01	1.0000E+00	-7.7901E+01	5.3344E+00	4.1462E+00	
3.2669E+01	-5.1062E+01	1.3999E+01	-1.0528E+01	8.4000E+00		
-2.4143E+00	-7.7857E-01	1.0000E+00	-8.0222E+01	5.9879E+00	3.5947E+00	
2.8714E+01	-4.1716E+01	1.2128E+01	-1.1513E+01	8.4500E+00		
-2.1571E+00	-7.1429E-01	1.0000E+00	-8.2066E+01	6.5459E+00	2.9974E+00	
2.4655E+01	-3.1985E+01	1.0179E+01	-1.2356E+01	8.5000E+00		
-1.9000E+00	-6.5000E-01	1.0000E+00	-8.3416E+01	7.0047E+00	2.3616E+00	
2.0516E+01	-2.1957E+01	8.1667E+00	-1.3053E+01	8.5500E+00		
-1.6429E+00	-5.8571E-01	1.0000E+00	-8.4259E+01	7.3618E+00	1.6947E+00	
1.6322E+01	-1.1719E+01	6.1093E+00	-1.3601E+01	8.6000E+00		
-1.3857E+00	-5.2143E-01	1.0000E+00	-8.4586E+01	7.6152E+00	1.0041E+00	
1.2099E+01	-1.3592E+00	4.0228E+00	-1.3997E+01	8.6500E+00		
-1.1286E+00	-4.5714E-01	1.0000E+00	-8.4394E+01	7.7639E+00	2.9759E-01	
7.8726E+00	9.0339E+00	1.9241E+00	-1.4241E+01	8.7000E+00		
-8.7143E-01	-3.9286E-01	1.0000E+00	-8.3683E+01	7.8077E+00	-4.1732E-01	
3.6687E+00	1.9374E+01	-1.7034E-01	-1.4332E+01	8.7500E+00		
-6.1429E-01	-3.2857E-01	1.0000E+00	-8.2459E+01	7.7472E+00	-1.1330E+00	
-4.8689E-01	2.9575E+01	-2.2441E+00	-1.4272E+01	8.8000E+00		
-3.5714E-01	-2.6429E-01	1.0000E+00	-8.0730E+01	7.5839E+00	-1.8420E+00	
-4.5686E+00	3.9555E+01	-4.2812E+00	-1.4064E+01	8.8500E+00		
-1.0000E-01	-2.0000E-01	1.0000E+00	-7.8509E+01	7.3199E+00	-2.5369E+00	
-8.5514E+00	4.9235E+01	-6.2662E+00	-1.3711E+01	8.9000E+00		

APPENDIX B – Continued

-9.5455E-02	-1.9091E-01	1.0000E+00	-7.5870E+01	6.9707E+00	-3.2110E+00
-1.2412E+01	5.6250E+01	-7.6922E+00	-1.3235E+01	8.9500E+00	
-9.0909E-02	-1.8182E-01	1.0000E+00	-7.2890E+01	6.5519E+00	-3.8588E+00
-1.6133E+01	6.2867E+01	-9.0462E+00	-1.2659E+01	9.0000E+00	
-8.6364E-02	-1.7273E-01	1.0000E+00	-6.9591E+01	6.0675E+00	-4.4753E+00
-1.9696E+01	6.9036E+01	-1.0318E+01	-1.1987E+01	9.0500E+00	
-8.1818E-02	-1.6364E-01	1.0000E+00	-6.5995E+01	5.5216E+00	-5.0561E+00
-2.3087E+01	7.4713E+01	-1.1500E+01	-1.1228E+01	9.1000E+00	
-7.7273E-02	-1.5455E-01	1.0000E+00	-6.2128E+01	4.9191E+00	-5.5967E+00
-2.6291E+01	7.9855E+01	-1.2582E+01	-1.0387E+01	9.1500E+00	
-7.2727E-02	-1.4545E-01	1.0000E+00	-5.8019E+01	4.2652E+00	-6.0935E+00
-2.9296E+01	8.4426E+01	-1.3558E+01	-9.4723E+00	9.2000E+00	
-6.8182E-02	-1.3636E-01	1.0000E+00	-5.3696E+01	3.5653E+00	-6.5429E+00
-3.2089E+01	8.8396E+01	-1.4421E+01	-8.4924E+00	9.2500E+00	
-6.3636E-02	-1.2727E-01	1.0000E+00	-4.9190E+01	2.8251E+00	-6.9418E+00
-3.4662E+01	9.1738E+01	-1.5166E+01	-7.4556E+00	9.3000E+00	
-5.9091E-02	-1.1818E-01	1.0000E+00	-4.4533E+01	2.0507E+00	-7.2876E+00
-3.7006E+01	9.4432E+01	-1.5789E+01	-6.3706E+00	9.3500E+00	
-5.4545E-02	-1.0909E-01	1.0000E+00	-3.9758E+01	1.2483E+00	-7.5782E+00
-3.9113E+01	9.6461E+01	-1.6285E+01	-5.2467E+00	9.4000E+00	
-5.0000E-02	-1.0000E-01	1.0000E+00	-3.4898E+01	4.2431E-01	-7.8118E+00
-4.0980E+01	9.7817E+01	-1.6654E+01	-4.0930E+00	9.4500E+00	
-4.5455E-02	-9.0909E-02	1.0000E+00	-2.9987E+01	-4.1489E-01	-7.9871E+00
-4.2602E+01	9.8493E+01	-1.6893E+01	-2.9190E+00	9.5000E+00	
-4.0909E-02	-8.1818E-02	1.0000E+00	-2.5060E+01	-1.2628E+00	-8.1035E+00
-4.3978E+01	9.8492E+01	-1.7002E+01	-1.7340E+00	9.5500E+00	
-3.6364E-02	-7.2727E-02	1.0000E+00	-2.0149E+01	-2.1129E+00	-8.1605E+00
-4.5109E+01	9.7819E+01	-1.6981E+01	-5.4755E-01	9.6000E+00	
-3.1818E-02	-6.3636E-02	1.0000E+00	-1.5289E+01	-2.9588E+00	-8.1584E+00
-4.5994E+01	9.6485E+01	-1.6834E+01	6.3121E-01	9.6500E+00	
-2.7273E-02	-5.4545E-02	1.0000E+00	-1.0511E+01	-3.7942E+00	-8.0977E+00
-4.6639E+01	9.4508E+01	-1.6561E+01	1.7931E+00	9.7000E+00	
-2.2727E-02	-4.5455E-02	1.0000E+00	-5.8485E+00	-4.6129E+00	-7.9795E+00
-4.7047E+01	9.1907E+01	-1.6168E+01	2.9294E+00	9.7500E+00	
-1.8182E-02	-3.6364E-02	1.0000E+00	-1.3307E+00	-5.4090E+00	-7.8053E+00
-4.7226E+01	8.8709E+01	-1.5658E+01	4.0315E+00	9.8000E+00	
-1.3636E-02	-2.7273E-02	1.0000E+00	3.0129E+00	-6.1768E+00	-7.5770E+00
-4.7183E+01	8.4944E+01	-1.5036E+01	5.0913E+00	9.8500E+00	
-9.0909E-03	-1.8182E-02	1.0000E+00	7.1548E+00	-6.9109E+00	-7.2970E+00
-4.6928E+01	8.0646E+01	-1.4310E+01	6.1011E+00	9.9000E+00	
-4.5455E-03	-9.0909E-03	1.0000E+00	1.1069E+01	-7.6062E+00	-6.9679E+00
-4.6472E+01	7.5854E+01	-1.3485E+01	7.0536E+00	9.9500E+00	
-5.6932E-13	-1.1386E-12	1.0000E+00	1.4733E+01	-8.2579E+00	-6.5927E+00
-4.5825E+01	7.0607E+01	-1.2570E+01	7.9422E+00	1.0000E+01	

Output Listing

THERE ARE 200 DATA POINTS, 10 ITERATIONS AND THE TIME INCREMENT IS .0500 SECONDS
THE SCALING PARAMETER, FINK'S VALUE IS .0100

MATRIX A HAS 4 ROWS AND 4 COLUMNS
-0. -0. -0. -0.
-0. -0. -0. -0.
-0. -1.0000E+00 -0. 7.8900E-02
1.0000E+00 -0. -0. -0.
MATRIX B HAS 4 ROWS AND 3 COLUMNS
-0. -0. -0.
-0. -0. -0.
-0. -0. -0.
-0. -0. -0.
MATRIX DIS HAS 1 ROWS AND 7 COLUMNS
1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00
1.0000E+0C 1.0000E+00
MATRIX F HAS 3 ROWS AND 4 COLUMNS
0. 0. 0. 0.
0. 0. 0. 0.
-0. -1.0000E+00 -0. 7.8900E-02
MATRIX G HAS 3 ROWS AND 3 COLUMNS
0. 0. 0.
0. 0. 0.
0. 0. 0.
MATRIX SZ HAS 1 ROWS AND 7 COLUMNS
1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+0C
1.0000E+00 1.0000E+00
MATRIX SU HAS 1 ROWS AND 3 COLUMNS
1.0000E+00 1.0000E+00 1.0000E+00
MATRIX D1 HAS 1 ROWS AND 7 COLUMNS
1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00
1.0000E+00 1.0000E+00

APPENDIX B – Continued

PARAMETER	GROUP	ROW	COLUMN	APRIORI DEVIATION	SCALE
1	1	1	1	-0.000099999.0000	1.0000
-0	5	1	1	-0.0000 -0.0000	1.0000
1	1	1	2	-0.000099999.0000	1.0000
-0	5	1	2	-0.0000 -0.0000	1.0000
1	1	1	3	-0.000099999.0000	1.0000
-0	5	1	3	-0.0000 -0.0000	1.0000
1	2	1	1	-0.000099999.0000	1.0000
-0	6	1	1	-0.0000 -0.0000	1.0000
-1	2	1	2	-0.000099999.0000	1.0000
-0	6	1	2	-0.0000 -0.0000	1.0000
-1	2	1	3	-0.000099999.0000	1.0000
-0	6	1	3	-0.0000 -0.0000	1.0000
1	1	2	1	-0.000099999.0000	1.0000
-0	5	2	1	-0.0000 -0.0000	1.0000
1	1	2	2	-0.000099999.0000	1.0000
-0	5	2	2	-0.0000 -0.0000	1.0000
1	1	2	3	-0.000099999.0000	1.0000
-0	5	2	3	-0.0000 -0.0000	1.0000
1	2	2	1	-0.000099999.0000	1.0000
-0	6	2	1	-0.0000 -0.0000	1.0000
1	2	2	2	-0.000099999.0000	1.0000
-0	6	2	2	-0.0000 -0.0000	1.0000
1	2	2	3	-0.000099999.0000	1.0000
-0	6	2	3	-0.0000 -0.0000	1.0000
1	1	3	1	-0.000099999.0000	1.0000
-0	5	3	1	-0.0000 -0.0000	1.0000
1	1	3	2	-0.000099999.0000	1.0000
-0	5	3	2	-0.0000 -0.0000	1.0000
1	2	3	3	-0.000099999.0000	1.0000
-0	6	3	3	-0.0000 -0.0000	1.0000
-1	-0	-0	-0	-0.0000 -0.0000	-0.0000

JKM = 16, NPM = 30

THE FIRST COLUMN IS TIME. THE NEXT 3 COLUMNS ARE CONTROL VARIABLES. THE NEXT 7 COLUMNS ARE RESPONSE VARIABLES.

TIME	CONTROL 1	CONTROL 2	CONTROL 3	RESPONSE 1	RESPONSE 2	RESPONSE 3	RESPONSE 4	RESPONSE 5	RESPONSE 6	RESPONSE 7
0.00	0.	0.	1.0000E+00	8.9372E-05	-4.1952E-05	2.2927E-05	2.2944E-06	1.6415E-03	-8.0941E-04	4.7933E-04
.05	0.	0.	1.0000E+00	1.6384E-04	-8.0879E-05	4.7857E-05	8.6888E-06	1.3345E-03	-7.4711E-04	5.1709E-04
.10	0.	0.	1.0000E+00	2.2256E-04	-1.1661E-04	7.4556E-05	1.8416E-05	1.0118E-03	-6.8167E-04	5.5005E-04
.15	0.	0.	1.0000E+00	2.6481E-04	-1.4900E-04	1.0278E-04	3.0670E-05	6.7548E-04	-6.1363E-04	5.7803E-04
.20	0.	0.	1.0000E+00	2.9005E-04	-1.7794E-04	1.3227E-04	4.4614E-05	3.3153E-04	-5.4357E-04	6.0087E-04
.25	0.	0.	1.0000E+00	2.9786E-04	-2.0334E-04	1.6278E-04	5.9385E-05	-1.9951E-05	-4.7208E-04	6.1847E-04
.30	0.	0.	1.0000E+00	2.8800E-04	-2.2513E-04	1.9403E-04	7.4105E-05	-3.7489E-04	-3.9974E-04	6.3076E-04
.35	0.	0.	1.0000E+00	2.6036E-04	-2.4331E-04	2.2577E-04	8.7888E-05	-7.3023E-04	-3.2714E-04	6.3773E-04
.40	0.	0.	1.0000E+00	2.1502E-04	-2.5785E-04	2.5772E-04	9.9846E-05	-1.0829E-03	-2.5484E-04	6.3940E-04
.45	0.	0.	1.0000E+00	1.5216E-04	-2.6880E-04	2.8962E-04	1.0910E-04	-1.4301E-03	-1.8342E-04	6.3581E-04
.50	0.	0.	1.0000E+00	7.2150E-05	-2.7622E-04	3.2121E-04	1.1478E-04	-1.7688E-03	-1.1342E-04	6.2708E-04
.55	0.	0.	1.0000E+00	-2.4530E-05	-2.8018E-04	3.5224E-04	1.1604E-04	-2.0963E-03	-4.5389E-05	6.1334E-04
.60	0.	0.	1.0000E+00	-1.3725E-04	-2.8080E-04	3.8247E-04	1.1206E-04	-2.4100E-03	2.0174E-05	5.9477E-04
.65	0.	0.	1.0000E+00	-2.6526E-04	-2.7821E-04	4.1164E-04	1.0206E-04	-2.7074E-03	8.2782E-05	5.7158E-04
.70	0.	0.	1.0000E+00	-4.0768E-04	-2.7258E-04	4.3955E-04	8.5290E-05	-2.9861E-03	1.4198E-04	5.4401E-04
.75	0.	0.	1.0000E+00	-5.6353E-04	-2.6408E-04	4.6598E-04	6.1064E-05	-3.2442E-03	1.9735E-04	5.1234E-04
.80	0.	5.0000E-01	1.0000E+00	5.0921E-02	-4.7040E-02	1.4466E-03	9.9991E-04	2.0536E+00	-1.8679E+00	5.1327E-02
.85	0.	1.0000E+00	1.0000E+00	2.0320E-01	-1.8650E-01	7.4774E-03	7.0519E-03	4.0194E+00	-3.7056E+00	2.0201E-01
.90	4.9000E+00	1.0000E+00	1.0000E+00	1.3997E+00	-5.0905E-01	2.6685E-02	4.0948E-02	4.3730E+01	-9.1812E+00	6.2049E-01
.95	9.8000E+00	1.0000E+00	1.0000E+00	4.5598E+00	-1.1024E+00	7.7664E-02	1.8395E-01	8.2525E+01	-1.4529E+01	1.4713E+00
1.00	9.5500E+00	1.0000E+00	1.0000E+00	8.5798E+00	-1.8129E+00	1.7739E+01	5.1324E+01	7.8187E+01	-1.3875E+01	2.5101E+00
1.05	9.3000E+00	1.0000E+00	1.0000E+00	1.2370E+01	-2.4881E+00	3.2771E+01	1.0379E+00	7.3338E+01	-1.3118E+01	3.4939E+00
1.10	2.5000E+00	1.0000E+00	1.0000E+00	1.4568E+01	-2.9374E+00	5.2013E+01	1.7207E+00	1.4618E+01	-4.8520E+00	4.1188E+00
1.15	-4.3000E+00	1.0000E+00	1.0000E+00	1.3844E+01	-2.9745E+00	7.2736E+01	2.4401E+00	-4.3445E+01	3.3565E+00	4.0874E+00
1.20	-4.3333E+00	1.0000E+00	1.0000E+00	1.1624E+01	-2.7959E+00	9.2251E+01	3.0772E+00	-4.5324E+01	3.7779E+00	3.7134E+00
1.25	-4.3667E+00	1.0000E+00	1.0000E+00	9.3161E+00	-2.5974E+00	1.0982E+00	3.6011E+00	-4.5956E+01	4.1528E+00	3.3093E+00
1.30	-4.4000E+00	1.0000E+00	1.0000E+00	6.9329E+00	-2.3815E+00	1.2530E+00	4.0076E+00	-4.8328E+01	4.4786E+00	2.8788E+00
1.35	-4.4333E+00	1.0000E+00	1.0000E+00	4.4878E+00	-2.1504E+00	1.3857E+00	4.2933E+00	-4.9431E+01	4.7533E+00	2.4256E+00
1.40	-4.4667E+00	1.0000E+00	1.0000E+00	1.9945E+00	-1.9070E+00	1.4952E+00	4.4555E+00	-5.0255E+01	4.9751E+00	1.9535E+00
1.45	-4.5000E+00	1.0000E+00	1.0000E+00	-5.3294E-01	-1.6538E+00	1.5808E+00	4.4922E+00	-5.0796E+01	5.1427E+00	1.4665E+00
1.50	-2.2000E+00	1.0000E+00	1.0000E+00	-2.6036E+00	-1.4598E+00	1.6437E+00	4.4108E+00	-3.2028E+01	2.6141E+00	1.0744E+00
1.55	1.0000E-01	1.0000E+00	1.0000E+00	-3.7379E+00	-1.3924E+00	1.6920E+00	4.2494E+00	-1.3358E+01	8.1551E-02	8.8490E-01
1.60	1.0000E-01	8.0833E-01	1.0000E+00	-4.4332E+00	-1.3681E+00	1.7337E+00	4.0453E+00	-1.4440E+01	8.8655E-01	7.7370E-01
1.65	1.0000E-01	6.1667E-01	1.0000E+00	-5.1803E+00	-1.3042E+00	1.7686E+00	3.8051E+00	-1.5428E+01	1.6687E+00	6.1776E-01
1.70	1.0000E-01	4.2500E-01	1.0000E+00	-5.9740E+00	-1.2017E+00	1.7946E+00	3.5264E+00	-1.6301E+01	2.4236E+00	4.1926E-01
1.75	1.0000E-01	2.3333E-01	1.0000E+00	-6.8080E+00	-1.0624E+00	1.8098E+00	3.2069E+00	-1.7035E+01	3.1470E+00	1.8068E-01
1.80	1.0000E-01	4.1667E-02	1.0000E+00	-7.6749E+00	-8.8766E+00	1.8120E+00	2.8450E+00	-1.7612E+01	3.8350E+00	-9.5236E-02
1.85	1.0000E-01	-1.5000E-01	1.0000E+00	-8.5662E+00	-6.7951E+00	1.7996E+00	2.4390E+00	-1.8012E+01	4.4843E+00	-4.0548E-01
1.90	1.0000E-01	-3.4167E-01	1.0000E+00	-9.4728E+00	-4.3995E+00	1.7709E+00	1.9880E+00	-1.8220E+01	5.0916E+00	-7.4683E-01
1.95	1.0000E-01	-5.3333E-01	1.0000E+00	-1.0385E+01	-1.7112E+01	1.7244E+00	1.4915E+00	-1.8223E+01	5.6542E+00	-1.1159E+00
2.00	1.0000E-01	-7.2500E-01	1.0000E+00	-1.1291E+01	1.2468E+01	1.6588E+00	9.4952E+00	-1.8008E+01	6.1699E+00	-1.5090E+00
2.05	1.0000E-01	-9.1667E-01	1.0000E+00	-1.2182E+01	4.4504E+01	1.5730E+00	3.6255E+00	-1.7566E+01	6.6366E+00	-1.9226E+00
2.10	1.0000E-01	-1.1083E+00	1.0000E+00	-1.3044E+01	7.8748E+01	1.4662E+00	-2.6828E+00	-1.6889E+01	7.0530E+00	-2.3527E+00
2.15	1.0000E-01	-1.3000E+00	1.0000E+00	-1.3867E+01	1.1495E+00	1.3375E+00	-9.4128E+00	-1.5973E+01	7.4180E+00	-2.7955E+00
2.20	1.0000E-01	-1.3000E+00	1.0000E+00	-1.4618E+01	1.5105E+00	1.1868E+00	-1.6538E+00	-1.4027E+01	7.0149E+00	-3.2276E+00
2.25	1.0000E-01	-1.3000E+00	1.0000E+00	-1.5266E+01	1.8503E+00	1.0153E+00	-2.4013E+00	-1.1873E+01	6.5712E+00	-3.6262E+00
2.30	1.0000E-01	-1.3000E+00	1.0000E+00	-1.5802E+01	2.1670E+00	8.2474E+00	-3.1785E+00	-9.5344E+00	6.0912E+00	-3.9887E+00
2.35	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6217E+01	2.4589E+00	6.1704E+00	-3.9795E+00	-7.0347E+00	5.5792E+00	-4.3128E+00
2.40	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6503E+01	2.7245E+00	3.9413E+00	-4.7980E+00	-4.3986E+00	5.0399E+00	-4.5965E+00
2.45	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6655E+01	2.9625E+00	1.5809E+00	-5.6275E+00	-1.6513E+00	4.4780E+00	-4.8382E+00
2.50	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6667E+01	3.1720E+00	-8.8965E+02	-6.4612E+00	1.1814E+00	3.8983E+00	-5.0366E+00
2.55	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6535E+01	3.3521E+00	-3.4484E+01	-7.2918E+00	4.0736E+00	3.3058E+00	-5.1908E+00
2.60	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6259E+01	3.5024E+00	-6.0730E+01	-8.1123E+00	6.9992E+00	2.7053E+00	-5.3002E+00
2.65	1.0000E-01	-1.3000E+00	1.0000E+00	-1.5835E+01	3.6226E+00	-8.7411E+01	-8.9152E+00	9.9323E+00	2.1019E+00	-5.3646E+00
2.70	1.0000E-01	-1.3000E+00	1.0000E+00	-1.5266E+01	3.7126E+00	-1.1430E+00	-9.6934E+00	1.2847E+01	1.5004E+00	-5.3840E+00
2.75	1.0000E-01	-1.3000E+00	1.0000E+00	-1.4551E+01	3.7727E+00	-1.4118E+00	-1.0439E+01	1.5719E+01	9.0566E-01	-5.3590E+00
2.80	1.0000E-01	-1.3000E+00	1.0000E+00	-1.3695E+01	3.8034E+00	-1.6782E+00	-1.1146E+01	1.8524E+01	3.2229E-01	-5.2904E+00
2.85	1.0000E-01	-1.3000E+00	1.0000E+00	-1.2701E+01	3.8052E+00	-1.9401E+00	-1.1807E+01	2.1237E+01	-2.4515E-01	-5.1794E+00
2.90	1.0000E-01	-1.3000E+00	1.0000E+00	-1.1573E+01	3.7792E+00	-2.1954E+00	-1.2414E+01	2.3837E+01	-7.9231E-01	-5.0272E+00
2.95	1.0000E-01	-1.3000E+00	1.0000E+00	-1.0319E+01	3.7264E+00	-2.4422E+00	-1.2962E+01	2.5303E+01	-1.3151E+00	-4.8359E+00

APPENDIX B – Continued

3.00	1.0000E-01	-1.3000E+00	1.0000E+00	-8.9454E+00	3.6482E+00	-2.6784E+00	-1.3444E+01	2.8615E+01	-1.8096E+00	-4.6072E+00
3.05	1.0000E-01	-1.3000E+00	1.0000E+00	-7.4605E+00	3.5460E+00	-2.9023E+00	-1.3855E+01	3.0754E+01	-2.2723E+00	-4.3437E+00
3.10	1.0000E-01	-1.3000E+00	1.0000E+00	-5.8732E+00	3.4215E+00	-3.1122E+00	-1.4188E+01	3.2705E+01	-2.6998E+00	-4.0477E+00
3.15	1.0000E-01	-1.3000E+00	1.0000E+00	-4.1934E+00	3.2766E+00	-3.3066E+00	-1.4440E+01	3.4451E+01	-3.0892E+00	-3.7220E+00
3.20	1.0000E-01	-1.3000E+00	1.0000E+00	-2.4317E+00	3.1133E+00	-3.4840E+00	-1.4606E+01	3.5981E+01	-3.4379E+00	-3.3696E+00
3.25	1.0000E-01	-1.3000E+00	1.0000E+00	-5.9913E-01	2.9336E+00	-3.6432E+00	-1.4682E+01	3.7282E+01	-3.7435E+00	-2.9937E+00
3.30	1.0000E-01	-1.3000E+00	1.0000E+00	1.2926E+00	2.7397E+00	-3.7830E+00	-1.4665E+01	3.8346E+01	-4.0042E+00	-2.5976E+00
3.35	1.0000E-01	-1.3000E+00	1.0000E+00	3.2314E+00	2.5339E+00	-3.9026E+00	-1.4552E+01	3.9165E+01	-4.2183E+00	-2.1841E+00
3.40	1.0000E-01	-1.3000E+00	1.0000E+00	5.2049E+00	2.3186E+00	-4.0012E+00	-1.4341E+01	3.9735E+01	-4.3849E+00	-1.7574E+00
3.45	1.0000E-01	-1.3000E+00	1.0000E+00	7.2006E+00	2.0962E+00	-4.0782E+00	-1.4031E+01	4.0052E+01	-4.5030E+00	-1.3206E+00
3.50	1.0000E-01	-1.3000E+00	1.0000E+00	9.2059E+00	1.8692E+00	-4.1332E+00	-1.3621E+01	4.0115E+01	-4.5724E+00	-8.7740E-01
3.55	1.0000E-01	-1.3000E+00	1.0000E+00	1.1208E+01	1.6398E+00	-4.1659E+00	-1.3111E+01	3.9925E+01	-4.5931E+00	-4.3131E-01
3.60	1.0000E-01	-1.3000E+00	1.0000E+00	1.3194E+01	1.4106E+00	-4.1763E+00	-1.2501E+01	3.9485E+01	-4.5654E+00	1.4138E-02
3.65	1.0000E-01	-1.3000E+00	1.0000E+00	1.5152E+01	1.1841E+00	-4.1645E+00	-1.1792E+01	3.8801E+01	-4.4902E+00	4.5547E-01
3.70	1.0000E-01	-1.3000E+00	1.0000E+00	1.7070E+01	9.6240E-01	-4.1309E+00	-1.0986E+01	3.7878E+01	-4.3685E+00	8.8929E-01
3.75	1.0000E-01	-1.3000E+00	1.0000E+00	1.8936E+01	7.4796E-01	-4.0758E+00	-1.0086E+01	3.6725E+01	-4.2019E+00	1.3123E+00
3.80	1.0000E-01	-1.3000E+00	1.0000E+00	2.0739E+01	5.4293E-01	-3.9999E+00	-9.0935E+00	3.5353E+01	-3.9922E+00	1.7213E+00
3.85	1.0000E-01	-1.3000E+00	1.0000E+00	2.2468E+01	3.4943E-01	-3.9039E+00	-8.0130E+00	3.3773E+01	-3.7414E+00	2.1134E+00
3.90	1.0000E-01	-1.3000E+00	1.0000E+00	2.4113E+01	1.6943E-01	-3.7889E+00	-6.8481E+00	3.2000E+01	-3.4521E+00	2.4856E+00
3.95	1.0000E-01	-1.3000E+00	1.0000E+00	2.5665E+01	4.8128E-03	-3.6557E+00	-5.6033E+00	3.0047E+01	-3.1270E+00	2.8353E+00
4.00	1.0000E-01	-1.3000E+00	1.0000E+00	2.7115E+01	-1.4271E-01	-3.5058E+00	-4.2833E+00	2.7930E+01	-2.7689E+00	3.1601E+00
4.05	1.0000E-01	-1.3000E+00	1.0000E+00	2.8456E+01	-2.7158E-01	-3.3402E+00	-2.8936E+00	2.5667E+01	-2.3810E+00	3.4577E+00
4.10	1.0000E-01	-1.3000E+00	1.0000E+00	2.9680E+01	-3.8038E-01	-3.1605E+00	-1.4397E+00	2.3276E+01	-1.9668E+00	3.7262E+00
4.15	1.0000E-01	-1.3000E+00	1.0000E+00	3.0781E+01	-4.6788E-01	-2.9681E+00	7.2358E-02	2.0776E+01	-1.5297E+00	3.9638E+00
4.20	1.0000E-01	-1.3000E+00	1.0000E+00	3.1756E+01	-5.3302E-01	-2.7646E+00	1.6363E+00	1.8185E+01	-1.0733E+00	4.1692E+00
4.25	1.0000E-01	-1.3000E+00	1.0000E+00	3.2599E+01	-5.7495E-01	-2.5517E+00	3.2458E+00	1.5525E+01	-6.0151E-01	4.3410E+00
4.30	1.0000E-01	-1.3000E+00	1.0000E+00	3.3307E+01	-5.9298E-01	-2.3311E+00	4.8940E+00	1.2814E+01	-1.1806E-01	4.4784E+00
4.35	1.0000E-01	-1.3000E+00	1.0000E+00	3.3880E+01	-5.8663E-01	-2.1045E+00	6.5742E+00	1.0074E+01	3.7315E-01	4.5807E+00
4.40	1.0000E-01	-1.3000E+00	1.0000E+00	3.4315E+01	-5.5560E-01	-1.8736E+00	8.2797E+00	7.3254E+00	8.6823E-01	4.6476E+00
4.45	1.0000E-01	-1.3000E+00	1.0000E+00	3.4613E+01	-4.9981E-01	-1.6403E+00	1.0000E+01	4.5875E+00	1.3633E+00	4.6790E+00
4.50	1.0000E-01	-1.3000E+00	1.0000E+00	3.4774E+01	-4.1933E-01	-1.4063E+00	1.1739E+01	1.8806E+00	1.8546E+00	4.6752E+00
4.55	1.0000E-01	-1.3000E+00	1.0000E+00	3.4801E+01	-3.1447E-01	-1.1734E+00	1.3479E+01	7.7582E-01	2.3383E+00	4.6365E+00
4.60	1.0000E-01	-1.3000E+00	1.0000E+00	3.4698E+01	-1.8569E-01	-9.4323E-01	1.5217E+01	-3.3630E+00	2.8108E+00	4.5637E+00
4.65	1.0000E-01	-1.3000E+00	1.0000E+00	3.4467E+01	-3.3644E-02	-7.1755E-01	1.6946E+01	-5.8629E+00	3.2685E+00	4.4579E+00
4.70	1.0000E-01	-1.3000E+00	1.0000E+00	3.4113E+01	1.4086E-01	-4.9797E-01	1.8661E+01	-8.2584E+00	3.7083E+00	4.3202E+00
4.75	1.0000E-01	-1.3000E+00	1.0000E+00	3.3643E+01	3.3683E-01	-2.8604E+01	2.0356E+01	-1.0533E+01	4.1268E+00	4.1521E+00
4.80	1.0000E-01	-1.3000E+00	1.0000E+00	3.3062E+01	5.5313E-01	-8.3234E-02	2.2024E+01	-1.2673E+01	4.5211E+00	3.9553E+00
4.85	1.0000E-01	-1.3000E+00	1.0000E+00	3.2378E+01	7.8849E-01	1.0905E-01	2.3660E+01	-1.4663E+01	4.8885E+00	3.7317E+00
4.90	1.0000E-01	-1.3000E+00	1.0000E+00	3.1598E+01	1.0415E+00	2.8953E-01	2.5260E+01	-1.6491E+01	5.2265E+00	3.4834E+00
4.95	1.0000E-01	-1.3000E+00	1.0000E+00	3.0732E+01	1.3106E+00	4.5702E-02	2.6818E+01	-1.8146E+01	5.5328E+00	3.2126E+00
5.00	-2.3500E+00	-1.3000E+00	1.0000E+00	2.9286E+01	1.6637E+00	6.0836E-01	2.8322E+01	-3.9592E+01	8.5789E+00	2.8105E+00
5.05	-4.8000E+00	-1.3000E+00	1.0000E+00	2.6783E+01	2.1670E+00	7.3364E-01	2.9727E+01	-6.0442E+01	1.1536E+01	2.1709E+00
5.10	-7.2500E+00	-1.3000E+00	1.0000E+00	2.3255E+01	2.8153E+00	8.2115E-01	3.0981E+01	-8.0572E+01	1.4379E+01	1.3010E+00
5.15	-9.7000E+00	-1.3000E+00	1.0000E+00	1.8740E+01	3.6025E+00	8.5959E-01	3.2034E+01	-9.9863E+01	1.7086E+01	2.1011E-01
5.20	-9.5000E+00	-9.0000E-01	1.0000E+00	1.3868E+01	4.4086E+00	8.4121E-01	3.2849E+01	-9.4956E+01	1.5140E+01	-9.3039E-01
5.25	-9.3000E+00	-5.0000E-01	1.0000E+00	9.2538E+00	5.1149E+00	7.6876E-01	3.3426E+01	-8.9510E+01	1.3098E+01	-1.9513E+00
5.30	-7.9000E+00	-5.0000E-01	1.0000E+00	5.1280E+00	5.7205E+00	6.4683E-01	3.3783E+01	-7.5464E+01	1.1101E+01	-2.8332E+00
5.35	-6.5000E+00	-5.0000E-01	1.0000E+00	1.7113E+00	6.2248E+00	4.8833E-01	3.3952E+01	-6.1161E+01	9.0563E+00	-3.5574E+00
5.40	-5.1000E+00	-5.0000E-01	1.0000E+00	-9.8575E-01	6.6253E+00	2.9585E-01	3.3967E+01	-4.6695E+01	6.9558E+00	-4.1199E+00
5.45	-3.7000E+00	-5.0000E-01	1.0000E+00	-2.9574E+00	6.9200E+00	7.9348E-02	3.3866E+01	-3.2160E+01	4.8261E+00	-4.5179E+00
5.50	-2.3000E+00	-5.0000E-01	1.0000E+00	-4.2025E+00	7.1078E+00	-1.5292E-01	3.3685E+01	-1.7652E+01	2.6853E+00	-4.7503E+00
5.55	-9.0000E-01	-5.0000E-01	1.0000E+00	-4.7249E+00	7.1887E+00	-3.9265E-01	3.3459E+01	-3.2624E+00	5.5161E-01	-4.8168E+00
5.60	5.0000E-01	-5.0000E-01	1.0000E+00	-4.5326E+00	7.1634E+00	-6.3158E-01	3.3225E+01	1.0915E+01	-1.5574E+00	-4.7187E+00
5.65	1.9000E+00	-5.0000E-01	1.0000E+00	-3.6387E+00	7.0337E+00	-8.6155E-01	3.3019E+01	2.4792E+01	-3.6242E+00	-4.4587E+00
5.70	3.3000E+00	-5.0000E-01	1.0000E+00	-2.0603E+00	6.8020E+00	-1.0745E+00	3.2874E+01	3.8281E+01	-5.6322E+00	-4.0403E+00
5.75	4.7000E+00	-5.0000E-01	1.0000E+00	1.8122E-01	6.4717E+00	-1.2628E+00	3.2825E+01	5.1300E+01	-7.5649E+00	-3.4686E+00
5.80	6.1000E+00	-5.0000E-01	1.0000E+00	3.0602E+00	6.0470E+00	-1.4187E+00	3.2904E+01	6.3770E+01	-9.4072E+00	-2.7496E+00
5.85	7.5000E+00	-5.0000E-01	1.0000E+00	6.5475E+00	5.5328E+00	-1.5351E+00	3.3142E+01	7.5618E+01	-1.1144E+01	-1.8904E+00
5.90	8.9000E+00	-5.0000E-01	1.0000E+00	1.0610E+01	4.9346E+00	-1.6053E+00	3.3570E+01	8.6775E+01	-1.2763E+01	-8.9924E-01
5.95	1.0300E+01	-5.0000E-01	1.0000E+00	1.5212E+01	4.2588E+00	-1.6227E+00	3.4214E+01	9.7179E+01	-1.4250E+01	2.1490E-01
6.00	1.0400E+01	-5.0000E-01	1.0000E+00	2.0049E+01	3.5489E+00	-1.5827E+00	3.5095E+01	9.6176E+01	-1.4123E+01	1.3832E+00
6.05	1.0500E+01	-5.0000E-01	1.0000E+00	2.4819E+01	2.8485E+00	-1.4846E+00	3.6218E+01	9.4529E+01	-1.3872E+01	2.5371E+00

APPENDIX B – Continued

6.10	1.0600E+01	-5.0000E-01	1.0000E+00	2.9491E+01	2.1637E+00	-1.3294E+00	3.7576E+01	9.2258E+01	-1.3501E+01	3.6680E+00
6.15	1.0700E+01	-5.0000E-01	1.0000E+00	3.4035E+01	1.5004E+00	-1.1183E+00	3.9165E+01	8.9388E+01	-1.3013E+01	4.7673E+00
6.20	9.5000E+00	-5.0000E-01	1.0000E+00	3.8155E+01	9.0115E-01	-8.5440E-01	4.0972E+01	7.5349E+01	-1.0942E+01	5.7679E+00
6.25	8.3000E+00	-5.0000E-01	1.0000E+00	4.1565E+01	4.0744E-01	-5.4451E-01	4.2967E+01	6.0987E+01	-8.7943E+00	6.6045E+00
6.30	7.1000E+00	-5.0000E-01	1.0000E+00	4.4250E+01	2.2651E-02	-1.9700E-01	4.5115E+01	4.6402E+01	-6.5886E+00	7.2722E+00
6.35	5.9000E+00	-5.0000E-01	1.0000E+00	4.6203E+01	-2.5080E-01	1.7961E-01	4.7379E+01	3.1692E+01	-4.3440E+00	7.7677E+00
6.40	4.7000E+00	-5.0000E-01	1.0000E+00	4.7419E+01	-4.1144E-01	5.7664E-01	4.9722E+01	1.6959E+01	-2.0794E+00	8.0891E+00
6.45	3.5000E+00	-5.0000E-01	1.0000E+00	4.7900E+01	-4.5876E-01	9.8538E-01	5.2108E+01	2.3003E+00	1.8606E-01	8.2358E+00
6.50	2.3000E+00	-5.0000E-01	1.0000E+00	4.7652E+01	-3.9317E-01	1.3971E+00	5.4499E+01	-1.2185E+01	2.4335E+00	8.2086E+00
6.55	1.1000E+00	-5.0000E-01	1.0000E+00	4.6687E+01	-2.1604E-01	1.8032E+00	5.6860E+01	-2.6403E+01	4.6445E+00	8.0097E+00
6.60	-1.0000E-01	-5.0000E-01	1.0000E+00	4.5018E+01	7.0346E-02	2.1950E+00	5.9155E+01	-4.0259E+01	6.8012E+00	7.6425E+00
6.65	-1.3000E+00	-5.0000E-01	1.0000E+00	4.2668E+01	4.6284E-01	2.5645E+00	6.1349E+01	-5.3666E+01	8.8860E+00	7.1117E+00
6.70	-2.5000E+00	-5.0000E-01	1.0000E+00	3.9661E+01	9.5745E-01	2.9034E+00	6.3410E+01	-6.6539E+01	1.0883E+01	6.4233E+00
6.75	-3.7000E+00	-5.0000E-01	1.0000E+00	3.6025E+01	1.5493E+00	3.2040E+00	6.5304E+01	-7.8798E+01	1.2775E+01	5.5844E+00
6.80	-3.7000E+00	9.0000E-01	1.0000E+00	3.21E3E+01	2.0679E+00	3.4629E+00	6.7009E+01	-7.4825E+01	7.9600E+00	4.7999E+00
6.85	-3.7000E+00	2.3000E+00	1.0000E+00	2.8547E+01	2.3451E+00	3.6889E+00	6.8526E+01	-7.0550E+01	3.1239E+00	4.2695E+00
6.90	-3.7000E+00	3.7000E+00	1.0000E+00	2.5130E+01	2.3805E+00	3.8946E+00	6.9868E+01	-6.6119E+01	-1.7055E+00	3.9914E+00
6.95	-3.7000E+00	5.1000E+00	1.0000E+00	2.1935E+01	2.1752E+00	4.0928E+00	7.1044E+01	-6.1677E+01	-6.5011E+00	3.9620E+00
7.00	-3.7000E+00	5.0900E+00	1.0000E+00	1.8815E+01	1.8635E+00	4.2928E+00	7.2063E+01	-6.3164E+01	-5.9687E+00	4.0321E+00
7.05	-3.7000E+00	5.0800E+00	1.0000E+00	1.5619E+01	1.5784E+00	4.4952E+00	7.2924E+01	-6.4655E+01	-5.4368E+00	4.0562E+00
7.10	-3.7000E+00	5.0700E+00	1.0000E+00	1.2349E+01	1.3197E+00	4.6977E+00	7.3623E+01	-6.6125E+01	-4.9104E+00	4.0348E+00
7.15	-3.7000E+00	5.0600E+00	1.0000E+00	9.0073E+00	1.0872E+00	4.8979E+00	7.4158E+01	-6.7549E+01	-4.3942E+00	3.9691E+00
7.20	-3.7000E+00	5.0500E+00	1.0000E+00	5.5957E+00	8.8004E-01	5.0938E+00	7.4523E+01	-6.8902E+01	-3.8929E+00	3.8601E+00
7.25	-3.7000E+00	5.0400E+00	1.0000E+00	2.1187E+00	6.9754E-01	5.2832E+00	7.4716E+01	-7.0161E+01	-3.4109E+00	3.7096E+00
7.30	-3.7000E+00	5.0300E+00	1.0000E+00	-1.4185E+00	5.3856E-01	5.4641E+00	7.4734E+01	-7.1306E+01	-2.9524E+00	3.5194E+00
7.35	-3.7000E+00	5.0200E+00	1.0000E+00	-5.0096E+00	4.0184E-01	5.6346E+00	7.4573E+01	-7.2315E+01	-2.5213E+00	3.2917E+00
7.40	-3.7000E+00	5.0100E+00	1.0000E+00	-8.6474E+00	2.8591E-01	5.7927E+00	7.4232E+01	-7.3170E+01	-2.1212E+00	3.0290E+00
7.45	-3.7000E+00	5.0000E+00	1.0000E+00	-1.2324E+00	1.8915E-01	5.9369E+00	7.3708E+01	-7.3854E+01	-1.7554E+00	2.7338E+00
7.50	-3.7000E+00	4.9900E+00	1.0000E+00	-1.6030E+00	1.0975E-01	6.0656E+00	7.2999E+01	-7.4353E+01	-1.4269E+00	2.4092E+00
7.55	-3.7000E+00	4.9800E+00	1.0000E+00	-1.9756E+00	4.5794E-02	6.1774E+00	7.2105E+01	-7.4652E+01	-1.1383E+00	2.0581E+00
7.60	-3.7000E+00	4.9700E+00	1.0000E+00	-2.3491E+00	-4.7846E-03	6.2710E+00	7.1023E+01	-7.4740E+01	-8.9197E-01	1.6838E+00
7.65	-3.7000E+00	4.9600E+00	1.0000E+00	-2.7226E+00	-4.4138E-02	6.3454E+00	6.9755E+01	-7.4610E+01	-6.8964E-01	1.2897E+00
7.70	-3.7000E+00	4.9500E+00	1.0000E+00	-3.0949E+00	-7.4509E-02	6.3997E+00	6.8301E+01	-7.4252E+01	-5.3285E-01	8.7925E-01
7.75	-3.7000E+00	4.9400E+00	1.0000E+00	-3.4647E+00	-9.8201E-02	6.4331E+00	6.6661E+01	-7.3663E+01	-4.2267E-01	4.5601E-01
7.80	-3.7000E+00	4.9300E+00	1.0000E+00	-3.8311E+00	-1.1756E-01	6.4452E+00	6.4837E+01	-7.2839E+01	-3.5980E-01	2.3589E-02
7.85	-3.7000E+00	4.9200E+00	1.0000E+00	-4.1927E+00	-1.3497E-01	6.4354E+00	6.2831E+01	-7.1780E+01	-3.4452E-01	-4.1438E-01
7.90	-3.7000E+00	4.9100E+00	1.0000E+00	-4.5485E+00	-1.5281E-01	6.4037E+00	6.0645E+01	-7.0486E+01	-3.7673E-01	-8.5428E-01
7.95	-3.7000E+00	4.9000E+00	1.0000E+00	-4.8972E+00	-1.7343E-01	6.3500E+00	5.8283E+01	-6.8961E+01	-4.5595E+00	-1.2925E+00
8.00	-3.7000E+00	3.4000E+00	1.0000E+00	-5.2531E+00	-1.9748E-02	6.2717E+00	5.5746E+01	-7.3341E+01	-4.9857E+00	-1.8771E+00
8.05	-3.7000E+00	1.9000E+00	1.0000E+00	-5.6298E+00	3.2276E-01	6.1568E+00	5.3026E+01	-7.7231E+01	-1.0292E+01	-2.7507E+00
8.10	-3.7000E+00	4.0000E+00	1.0000E+00	-6.0244E+00	9.6659E-01	5.9914E+00	5.0113E+01	-8.0479E+01	1.5434E+01	-3.8997E+00
8.15	-3.7000E+00	-1.1000E+00	1.0000E+00	-6.4332E+00	1.8628E+00	5.7619E+00	4.6999E+01	-8.2941E+01	2.0384E+01	-5.3088E+00
8.20	-3.4429E+00	-1.0357E+00	1.0000E+00	-6.8308E+00	2.8476E+00	5.4590E+00	4.3681E+01	-7.5956E+01	1.8982E+01	-6.7898E+00
8.25	-3.1857E+00	-9.7143E-01	1.0000E+00	-7.1916E+00	3.7587E+00	5.0848E+00	4.0174E+01	-6.8263E+01	1.7441E+01	-8.1593E+00
8.30	-2.9268E+00	-9.0714E-01	1.0000E+00	-7.5124E+00	4.5896E+00	4.6452E+00	3.6497E+01	-7.3341E+01	4.9857E+00	-1.8771E+00
8.35	-2.6714E+00	-8.4286E-01	1.0000E+00	-7.9701E+00	5.3344E+00	4.1462E+00	3.2669E+01	-7.7231E+01	1.0292E+01	-2.7507E+00
8.40	-2.4143E+00	-7.7857E-01	1.0000E+00	-8.0222E+00	5.9879E+00	3.5947E+00	2.8714E+01	-4.1716E+01	1.2128E+01	-1.1513E+01
8.45	-2.1571E+00	-7.1429E-01	1.0000E+00	-8.2066E+00	6.5459E+00	2.9974E+00	2.4655E+01	-3.1985E+01	1.0179E+01	-1.2356E+01
8.50	-1.9000E+00	-6.5000E-01	1.0000E+00	-8.3416E+00	7.0047E+00	2.3616E+00	2.0516E+01	-2.1957E+01	8.1667E+00	-1.3053E+01
8.55	-1.6429E+00	-5.8571E-01	1.0000E+00	-8.4259E+00	7.3618E+00	1.6947E+00	1.6322E+01	-1.7179E+01	6.1093E+00	-1.3601E+01
8.60	-1.3857E+00	-5.2143E-01	1.0000E+00	-8.4586E+00	7.6152E+00	1.0041E+00	1.2099E+01	-1.3592E+01	4.0228E+00	-1.3997E+01
8.65	-1.1286E+00	-4.5714E-01	1.0000E+00	-8.4394E+00	7.7639E+00	2.9759E-01	7.8726E+00	9.0339E+00	1.9241E+00	-1.4241E+01
8.70	-8.7143E-01	-3.9286E-01	1.0000E+00	-8.3683E+00	7.8077E+00	-4.1732E-01	3.6687E+00	1.9374E+00	-1.7034E-01	-1.4332E+01
8.75	-6.1429E-01	-3.2857E-01	1.0000E+00	-8.2459E+00	7.7472E+00	-1.1330E+00	-4.8689E-01	2.9575E+00	-2.2441E+00	-1.4272E+01
8.80	-3.5714E-01	-2.6429E-01	1.0000E+00	-8.0730E+00	7.5839E+00	-1.8420E+00	-4.5686E+00	3.9555E+00	-4.2812E+00	-1.4064E+01
8.85	-1.0000E-01	-2.0000E-01	1.0000E+00	-7.8509E+00	7.3199E+00	-2.5369E+00	-8.5514E+00	4.9235E+00	-6.2662E+00	-1.3711E+01
8.90	-9.5455E-02	-1.9091E-01	1.0000E+00	-7.5870E+00	6.9707E+00	-3.2110E+00	-1.2412E+01	5.6250E+00	-7.6922E+00	-1.3235E+01
8.95	-9.0909E-02	-1.8182E-01	1.0000E+00	-7.2890E+00	6.5519E+00	-3.8588E+00	-1.6133E+01	6.2867E+00	-9.0462E+00	-1.2659E+01
9.00	-8.6364E-02	-1.7273E-01	1.0000E+00	-6.9591E+00	6.0675E+00	-4.4753E+00	-1.9696E+01	6.9036E+00	-1.0318E+01	-1.1987E+01
9.05	-8.1818E-02	-1.6364E-01	1.0000E+00	-6.5995E+00	5.5216E+00	-5.0561E+00	-2.3087E+01	7.4713E+00	-1.1500E+01	-1.1228E+01
9.10	-7.7273E-02	-1.5455E-01	1.0000E+00	-6.2128E+00	4.9191E+00	-5.5967E+00	-2.6291E+01	7.9855E+00	-1.2582E+01	-1.0387E+01
9.15	-7.2727E-02	-1.4545E-01	1.0000E+00	-5.8019E+00	4.2652E+00	-6.0935E+00	-2.9296E+01	8.4426E+00	-1.3558E+01	-9.4723E+00
9.20	-6.8182E-02	-1.3636E-01	1.0000E+00	-5.3696E+00	3.5653E+00	-6.5429E+00	-3.2089E+01	8.8396E+00	-1.4421E+01	-8.4924E+00

APPENDIX B – Continued

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9.25 -6.3636E-02 -1.2727E-01 1.0000E+00 -4.9190E+01 2.8251E+00 -6.9418E+00 -3.4662E+01 9.1738E+01 -1.5166E+01 -7.4556E+00
9.30 -5.9091E-02 -1.1818E-01 1.0000E+00 -4.4533E+01 2.0507E+00 -7.2876E+00 -3.7006E+01 9.4432E+01 -1.5789E+01 -6.3706E+00
9.35 -5.4545E-02 -1.0909E-01 1.0000E+00 -3.9758E+01 1.2483E+00 -7.5782E+00 -3.9113E+01 9.6461E+01 -1.6285E+01 -5.2467E+00
9.40 -5.0000E-02 -1.0000E-01 1.0000E+00 -3.4898E+01 4.2431E-01 -7.8118E+00 -4.0980E+01 9.7817E+01 -1.6654E+01 -4.0930E+00
9.45 -4.5455E-02 -9.0909E-02 1.0000E+00 -2.9987E+01 -4.1489E-01 -7.9871E+00 -4.2602E+01 9.8493E+01 -1.6893E+01 -2.9190E+00
9.50 -4.0909E-02 -8.1818E-02 1.0000E+00 -2.5060E+01 -1.2628E+00 -8.1035E+00 -4.3978E+01 9.8492E+01 -1.7002E+01 -1.7340E+00
9.55 -3.6364E-02 -7.2727E-02 1.0000E+00 -2.0149E+01 -2.1129E+00 -8.1605E+00 -4.5109E+01 9.7819E+01 -1.6981E+01 -5.4755E-01
9.60 -3.1818E-02 -6.3636E-02 1.0000E+00 -1.5289E+01 -2.5588E+00 -8.1584E+00 -4.5994E+01 9.6485E+01 -1.6834E+01 6.3121E-01
9.65 -2.7273E-02 -5.4545E-02 1.0000E+00 -1.0511E+01 -3.7942E+00 -8.0977E+00 -4.6639E+01 9.4508E+01 -1.6561E+01 1.7931E+00
9.70 -2.2727E-02 -4.5455E-02 1.0000E+00 -5.8485E+00 -4.6129E+00 -7.9795E+00 -4.7047E+01 9.1907E+01 -1.6168E+01 2.9294E+00
9.75 -1.8182E-02 -3.6364E-02 1.0000E+00 -1.3307E+00 -5.4090E+00 -7.8053E+00 -4.7226E+01 8.8709E+01 -1.5658E+01 4.0315E+00
9.80 -1.3636E-02 -2.7273E-02 1.0000E+00 3.0129E+00 -6.1768E+00 -7.5770E+00 -4.7183E+01 8.4944E+01 -1.5036E+01 5.0913E+00
9.85 -9.0909E-03 -1.8182E-02 1.0000E+00 7.1548E+00 -6.9109E+00 -7.2970E+00 -4.6928E+01 8.0646E+01 -1.4310E+01 6.1011E+00
9.90 -4.5455E-03 -9.0909E-03 1.0000E+00 1.1069E+01 -7.6062E+00 -6.9679E+00 -4.6472E+01 7.5854E+01 -1.3485E+01 7.0536E+00
9.95 -5.6932E-13 -1.1386E-12 1.0000E+00 1.4733E+01 -8.2579E+00 -6.5927E+00 -4.5825E+01 7.0607E+01 -1.2570E+01 7.9422E+00

THE VALUE OF THE FIT ERROR IS 5.19084E+03 FOR THE ITERATION NUMBER 0
MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
1.0968E+03 1.0882E+01 1.2861E+01 1.2049E+03 2.7745E+03
6.3254E+01 2.7582E+01

MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
3.3118E+01 3.2989E+00 3.5863E+00 3.4712E+01 5.2674E+01
7.9533E+00 5.2519E+00

MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
9.7537E+02 1.0513E+01 -2.2491E+02 2.1901E+02 -2.1106E+02
1.9261E+02 -1.0977E+01 3.1953E+01 2.0065E+01 -2.7480E+01
4.0500E+00 2.4171E+01 9.3946E+01 2.6661E-01 -2.5070E+00
5.1908E+03

WOULD YOU BELIEVE.....
1.5117555E-05

MATRIX DC HAS 1 ROWS AND 15 COLUMNS
-4.0576E-01 6.2752E-01 -5.5459E+00 9.6505E+00 -5.3042E+00
-1.2882E+00 6.1596E-02 -1.3353E-01 1.6400E+00 -1.5716E+00
-2.0338E+00 -8.2433E-02 3.3916E-02 8.2978E-03 -5.4547E-01
THE VALUE OF THE FIT ERROR IS 5.35909E+02 FOR THE ITERATION NUMBER 1
MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
8.1382E+01 8.0695E+00 2.4167E+00 2.5188E+01 3.9272E+02
1.9191E+01 6.9371E+00

MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
9.0212E+00 2.8407E+00 1.5546E+00 5.0188E+00 1.9817E+01
4.3807E+00 2.6338E+00

MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
7.4488E+01 -3.6222E+00 -6.2248E-02 -1.3093E+01 2.1587E+01
1.5661E+01 -7.4364E+02 3.9285E+01 5.5023E+01 2.9144E+01
-7.7703E+00 6.2957E+01 7.9773E+01 2.3470E+01 1.1470E+01
5.3591E+02

WOULD YOU BELIEVE.....
1.5985992E-04

MATRIX DC HAS 1 ROWS AND 15 COLUMNS
5.7770E-01 1.4666E+00 -3.8994E+00 -7.3725E-01 1.2303E+01
-1.6013E+00 -2.2427E-01 -7.7426E-01 -2.0399E-01 4.5331E-02
-1.8603E+00 1.0177E+00 6.0879E-02 -6.8057E-01 8.6466E-01
THE VALUE OF THE FIT ERROR IS 2.67663E+02 FOR THE ITERATION NUMBER 2
MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
3.6351E+01 3.9183E+00 3.7904E+00 1.4265E+02 7.2200E+01
5.1011E+00 3.6521E+00

MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
6.0292E+00 1.9795E+00 1.9469E+00 1.1944E+01 8.4971E+00
2.2586E+00 1.9111E+00

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MATRIX GRAC HAS 1 ROWS AND 16 COLUMNS
 -7.9885E+01 1.6397E+01 7.7558E+01 -3.6834E+01 2.5243E+01
 2.8230E+01 1.8387E+03 3.3001E+01 2.4093E+02 6.8572E+01
 3.6008E+01 1.4235E+02 -3.3650E+02 -3.6445E+02 -7.4581E+01
 2.6766E+02
 WOULD YOU BELIEVE.....
 7.6661394E-07

MATRIX DC HAS 1 ROWS AND 15 COLUMNS
 -4.8573E-01 -2.4042E+00 -2.1477E+C0 -7.5156E-01 -2.2474E+00
 4.0716E+00 1.7355E-01 9.0476E-01 8.5963E-01 4.1626E-01
 2.4093E-01 -8.6947E-01 -1.8133E-02 3.1898E-01 1.8371E-01
 THE VALUE OF THE FIT ERROR IS 1.11355E+02 FOR THE ITERATION NUMBER 3

MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
 1.5170E+01 1.9705E+00 1.2194E-01 7.8761E+01 1.4327E+01
 7.6415E-01 2.4093E-01

MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
 3.8949E+00 1.4037E+00 3.4920E-C1 8.8747E+00 3.7851E+00
 8.7416E-01 4.9084E-01

MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
 -3.1804E+C2 -8.8816E+01 -5.8652E+C0 1.4504E+01 -3.4808E+00
 -4.8215E+01 -1.6484E+03 -5.5075E+C2 3.2447E+01 -6.2763E+01
 4.3889E+01 -2.3004E+02 4.4747E+C0 2.0853E+01 4.2124E+01
 1.1136E+02
 WOULD YOU BELIEVE.....
 3.1164800E-04

MATRIX DC HAS 1 ROWS AND 15 COLUMNS
 3.7891E-02 3.1380E-01 1.0566E-C1 4.5832E-02 -5.5311E-01
 -8.0468E-01 -1.3262E-02 -1.8334E-01 -1.3272E-01 -2.9255E-02
 -7.7024E-02 -3.4351E-02 5.0111E-03 1.1390E-02 -4.0531E-01
 THE VALUE OF THE FIT ERROR IS 2.72967E+00 FOR THE ITERATION NUMBER 4

MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
 2.1043E-01 7.6456E-02 9.7859E-C4 2.3095E+00 1.0243E-01
 2.7835E-02 2.0422E-03

MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
 4.5872E-01 2.7651E-01 3.1282E-02 1.5197E+00 3.2005E-01
 1.6684E-01 4.5191E-02

MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
 -2.8990E+01 -1.0237E+01 4.7531E-C1 1.4653E+00 7.7312E-01
 -7.4411E+00 -1.6044E+02 -5.7421E+01 2.3224E+00 -3.1222E-01
 7.2239E+00 -3.7176E+01 1.25C5E+C0 2.0022E-01 1.1348E+01
 2.7297E+00
 WOULD YOU BELIEVE.....
 8.6634005E-05

MATRIX DC HAS 1 ROWS AND 15 COLUMNS
 2.1801E-03 5.8226E-02 9.4842E-C2 1.4850E-02 -2.5872E-02
 -2.7843E-01 3.3919E-03 -2.3006E-02 5.6675E-03 -2.7452E-04
 -3.0410E-02 -4.4265E-02 1.2779E-C3 2.4770E-02 -8.4584E-02
 THE VALUE OF THE FIT ERROR IS 4.41052E-03 FOR THE ITERATION NUMBER 5

MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
 6.7047E-04 2.2670E-04 9.2182E-05 4.5641E-04 2.6849E-03
 2.0698E-04 7.2870E-05

MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
 2.5893E-02 1.5057E-02 9.6012E-C3 2.1364E-02 5.1816E-02
 1.4387E-02 8.5364E-03

MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
 2.2989E-01 -9.2070E-02 -2.2273E-C1 1.9753E-01 -4.3745E-02
 -8.4119E-02 -9.0750E+00 -2.3197E-01 1.3694E-01 -5.8027E-01
 -1.5326E-01 -3.7310E-01 6.2925E+C0 1.2976E+00 1.3671E-01
 4.4105E-03
 WOULD YOU BELIEVE.....
 8.0870804E-05

MATRIX DC HAS 1 ROWS AND 15 COLUMNS
 -1.1106E-04 1.3073E-03 -6.5665E-C3 9.1840E-04 5.4555E-03
 -7.5666E-02 2.5109E-04 8.4918E-04 1.2352E-03 -1.0992E-03
 -1.0369E-03 8.6470E-03 6.2564E-05 1.8393E-03 -1.0767E-02
 THE VALUE OF THE FIT ERROR IS 1.02760E-04 FOR THE ITERATION NUMBER 6
 MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
 1.1862E-05 2.4446E-06 6.4610E-06 2.3854E-05 5.1881E-05
 4.2667E-06 1.9910E-06
 MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
 3.4441E-03 1.5635E-03 2.5418E-03 4.8841E-03 7.2029E-03
 2.0656E-03 1.4110E-03
 MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
 1.3120E-01 -1.3166E-02 -1.6781E-C2 6.5262E-03 -4.2880E-04
 -5.4349E-04 -7.8444E-01 7.8726E-C2 -3.6398E-02 1.1753E-03
 -4.3023E-02 7.7035E-03 -3.2960E-C1 1.8323E-01 -3.2993E-04
 1.0276E-04
 WOULD YOU BELIEVE.....
 8.3062154E-05
 MATRIX DC HAS 1 ROWS AND 15 COLUMNS
 -8.6889E-05 -1.4371E-04 -1.0037E-03 -9.8075E-05 -1.8201E-04
 -1.7278E-02 3.4672E-05 3.0734E-C4 1.7953E-04 2.7525E-05
 1.8921E-04 2.6055E-03 -9.0888E-06 2.8012E-04 -1.5325E-03
 THE VALUE OF THE FIT ERROR IS 2.29906E-05 FOR THE ITERATION NUMBER 7
 MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
 1.7275E-06 8.4306E-08 1.7895E-06 1.7192E-05 1.8254E-06
 2.2031E-07 1.5132E-07
 MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
 1.3143E-03 2.9036E-04 1.3377E-03 4.1464E-03 1.3511E-03
 4.6937E-04 3.8900E-04
 MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
 7.8532E-03 -1.0569E-03 -5.5453E-04 -4.0673E-04 1.5895E-04
 -2.5042E-04 -3.7550E-02 5.2844E-03 -4.7062E-03 1.9848E-03
 -2.8049E-03 1.2471E-03 -6.3983E-02 1.0544E-02 -1.3571E-05
 2.2991E-05
 WOULD YOU BELIEVE.....
 8.3433431E-05
 MATRIX DC HAS 1 ROWS AND 15 COLUMNS
 -3.6997E-05 -8.3164E-06 -2.4369E-C4 -2.9114E-06 -1.1388E-04
 -3.3925E-03 1.0085E-05 6.0567E-C5 6.0934E-05 1.7531E-05
 4.8660E-05 5.1169E-04 -4.9164E-06 5.8552E-05 -2.8182E-04
 THE VALUE OF THE FIT ERROR IS 2.05472E-05 FOR THE ITERATION NUMBER 8
 MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
 1.4983E-06 3.4114E-08 1.5057E-06 1.7077E-05 6.1100E-07
 5.9819E-08 1.6148E-07
 MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
 1.2241E-03 1.8470E-04 1.2271E-C3 4.1324E-03 7.8166E-04
 2.4458E-04 4.0184E-04
 MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
 5.9765E-C4 -6.2485E-05 9.5939E-06 -8.3012E-05 2.8771E-05
 -5.0517E-05 -1.5591E-04 5.0592E-04 -4.4072E-04 2.8803E-04
 -1.8818E-04 2.4321E-04 -7.6072E-C3 4.7133E-04 9.5277E-06
 2.0947E-05
 WOULD YOU BELIEVE.....
 8.3486853E-C5
 MATRIX DC HAS 1 ROWS AND 15 COLUMNS
 -8.9911E-06 -5.2281E-06 -7.0370E-C5 1.0552E-06 -3.1564E-05
 -6.5232E-04 2.3883E-06 1.3300E-05 1.8493E-05 3.4129E-06
 1.1277E-05 9.5692E-05 -1.4811E-C6 1.0794E-05 -5.7084E-05
 THE FIRST COLUMN IS TIME. THE NEXT 3 COLUMNS ARE CONTROL VARIABLES. THE NEXT 7 COLUMNS ARE RESPONSE VARIABLES.
 .05 0. 0. 1.0000E+00 1.7171E-04 -8.3609E-05 4.4524E-05 8.8780E-06 1.5094E-03 -8.0499E-04 4.5268E-04
 .10 0. 0. 1.0000E+00 2.4000E-04 -1.2239E-04 6.8122E-05 1.9231E-05 1.2192E-03 -7.4562E-04 4.9051E-04
 .15 0. 0. 1.0000E+00 2.9337E-04 -1.5812E-04 9.3498E-05 3.2629E-05 9.1306E-04 -6.8305E-04 5.2374E-04
 .20 0. 0. 1.0000E+00 3.3109E-04 -1.9065E-04 1.2042E-04 4.8307E-05 5.9381E-04 -6.1781E-04 5.5216E-04

APPENDIX B – Continued

.25	0.	0.	1.0000E+00	3.5258E-04	-2.1986E-04	1.4863E-04	6.5468E-05	2.6441E-04	-5.5047E-04	5.7563E-04
.30	0.	0.	1.0000E+00	3.5741E-04	-2.4567E-04	1.7789E-04	8.3288E-05	-7.2146E-05	-4.8159E-04	5.9402E-04
.35	0.	0.	1.0000E+00	3.4530E-04	-2.6801E-04	2.0795E-04	1.0093E-04	-4.1286E-04	-4.1174E-04	6.0728E-04
.40	0.	0.	1.0000E+00	3.1610E-04	-2.8684E-04	2.3854E-04	1.1753E-04	-7.5476E-04	-3.4148E-04	6.1537E-04
.45	0.	0.	1.0000E+00	2.6985E-04	-3.0216E-04	2.6940E-04	1.3225E-04	-1.0949E-03	-2.7138E-04	6.1829E-04
.50	0.	0.	1.0000E+00	2.0669E-04	-3.1399E-04	3.0028E-04	1.4424E-04	-1.4304E-03	-2.0199E-04	6.1610E-04
.55	0.	0.	1.0000E+00	1.2693E-04	-3.2238E-04	3.3092E-04	1.5264E-04	-1.7584E-03	-1.3384E-04	6.0888E-04
.60	0.	0.	1.0000E+00	3.1016E-05	-3.2740E-04	3.6109E-04	1.5666E-04	-2.0763E-03	-6.7472E-05	5.9676E-04
.65	0.	0.	1.0000E+00	-8.0488E-05	-3.2916E-04	3.9052E-04	1.5549E-04	-2.3815E-03	-3.3737E-06	5.7990E-04
.70	0.	0.	1.0000E+00	-2.0688E-04	-3.2779E-04	4.1900E-04	1.4836E-04	-2.6714E-03	5.7975E-05	5.5850E-04
.75	0.	0.	1.0000E+00	-3.4734E-04	-3.2342E-04	4.4630E-04	1.3456E-04	-2.9439E-03	1.1612E-04	5.3279E-04
.80	0.	5.0000E-01	1.0000E+00	5.1043E-02	-4.7042E-02	1.7455E-03	1.4086E-03	2.0538E+00	-1.8680E+00	5.1208E-02
.85	0.	1.0000E+00	1.0000E+00	2.0323E-01	-1.8645E-01	8.0880E-03	7.7910E-03	4.0197E+00	-3.7057E+00	2.0175E-01
.90	4.9000E+00	1.0000E+00	1.0000E+00	1.3980E+00	-5.0881E-01	2.8672E-02	4.7942E-02	4.3731E+01	-9.1814E+00	6.1996E-01
.95	9.8000E+00	1.0000E+00	1.0000E+00	4.5563E+00	-1.1019E+00	8.1014E-02	1.9711E-01	8.2525E+01	-1.4529E+01	1.4705E+00
1.00	9.5500E+00	1.0000E+00	1.0000E+00	8.5764E+00	-1.8125E+00	1.8063E-01	5.2590E-01	7.8188E+01	-1.3875E+01	2.5093E+00
1.05	9.3000E+00	1.0000E+00	1.0000E+00	1.2367E+01	-2.4877E+00	3.3084E-01	1.0501E+00	7.3338E+01	-1.3118E+01	3.4932E+00
1.10	2.5000E+00	1.0000E+00	1.0000E+00	1.4567E+01	-2.9373E+00	5.2129E-01	1.7240E+00	1.4618E+01	-4.8522E+00	4.1185E+00
1.15	-4.3000E+00	1.0000E+00	1.0000E+00	1.3846E+01	-2.9746E+00	7.2658E-01	2.4348E+00	-4.3444E+01	3.3563E+00	4.0875E+00
1.20	-4.3333E+00	1.0000E+00	1.0000E+00	1.1626E+01	-2.7961E+00	9.2172E-01	3.0719E+00	-4.5323E+01	3.7777E+00	3.7135E+00
1.25	-4.3667E+00	1.0000E+00	1.0000E+00	9.3179E+00	-2.5976E+00	1.0974E+00	3.5958E+00	-4.6956E+01	4.1527E+00	3.3095E+00
1.30	-4.4000E+00	1.0000E+00	1.0000E+00	6.9347E+00	-2.3816E+00	1.2522E+00	4.0023E+00	-4.8328E+01	4.4785E+00	2.8790E+00
1.35	-4.4333E+00	1.0000E+00	1.0000E+00	4.4896E+00	-2.1506E+00	1.3849E+00	4.2881E+00	-4.9430E+01	4.7532E+00	2.4258E+00
1.40	-4.4667E+00	1.0000E+00	1.0000E+00	1.9963E+00	-1.9072E+00	1.4944E+00	4.4504E+00	-5.0256E+01	4.9751E+00	1.9537E+00
1.45	-4.5000E+00	1.0000E+00	1.0000E+00	-5.3113E-01	-1.6540E+00	1.5800E+00	4.4871E+00	-5.0796E+01	5.1427E+00	1.4667E+00
1.50	-2.2000E+00	1.0000E+00	1.0000E+00	-2.6027E+00	-1.4599E+00	1.6436E+00	4.4087E+00	-3.2028E+01	2.6141E+00	1.0745E+00
1.55	1.0000E-01	1.0000E+00	1.0000E+00	-3.7378E+00	-1.3925E+00	1.6926E+00	4.2503E+00	-1.3358E+01	8.1541E-02	8.8488E-01
1.60	1.0000E-01	8.0833E-01	1.0000E+00	-4.4330E+00	-1.3682E+00	1.7341E+00	4.0461E+00	-1.4440E+01	8.8657E-01	7.7374E-01
1.65	1.0000E-01	6.1667E-01	1.0000E+00	-5.1801E+00	-1.3042E+00	1.7689E+00	3.8058E+00	-1.5429E+01	1.6687E+00	6.1786E-01
1.70	1.0000E-01	4.2500E-01	1.0000E+00	-5.9738E+00	-1.2018E+00	1.7948E+00	3.5270E+00	-1.6301E+01	2.4236E+00	4.1941E-01
1.75	1.0000E-01	2.3333E-01	1.0000E+00	-6.8078E+00	-1.0625E+00	1.8098E+00	3.2074E+00	-1.7035E+01	3.1470E+00	1.8088E-01
1.80	1.0000E-01	4.1667E-02	1.0000E+00	-7.6746E+00	-8.8779E-01	1.8120E+00	2.8453E+00	-1.7612E+01	3.8351E+00	-9.4981E-02
1.85	1.0000E-01	-1.5000E-01	1.0000E+00	-8.5660E+00	-6.7976E-01	1.7994E+00	2.4349E+00	-1.8012E+01	4.4843E+00	-4.0518E-01
1.90	1.0000E-01	-3.4167E-01	1.0000E+00	-9.4726E+00	-4.4012E-01	1.7706E+00	1.9881E+00	-1.8221E+01	5.0917E+00	-7.4648E-01
1.95	1.0000E-01	-5.3333E-01	1.0000E+00	-1.0384E+01	-1.7131E-01	1.7240E+00	1.4915E+00	-1.8223E+01	5.6543E+00	-1.1155E+00
2.00	1.0000E-01	-7.2500E-01	1.0000E+00	-1.1291E+01	-1.2447E-01	1.6583E+00	9.4941E-01	-1.8008E+01	6.1700E+00	-1.5086E+00
2.05	1.0000E-01	-9.1667E-01	1.0000E+00	-1.2181E+01	-4.4481E-01	1.5725E+00	3.6234E-01	-1.7566E+01	6.6368E+00	-1.9221E+00
2.10	1.0000E-01	-1.1083E+00	1.0000E+00	-1.3044E+01	-7.6724E-01	1.4665E+00	-2.6860E-01	-1.6889E+01	7.0531E+00	-2.3522E+00
2.15	1.0000E-01	-1.3000E+00	1.0000E+00	-1.3866E+01	-1.1492E+00	1.3367E+00	-9.4171E-01	-1.5974E+01	7.4182E+00	-2.7950E+00
2.20	1.0000E-01	-1.3000E+00	1.0000E+00	-1.4617E+01	-1.5102E+00	1.1860E+00	-1.6542E+00	-1.4027E+01	7.0151E+00	-3.2271E+00
2.25	1.0000E-01	-1.3000E+00	1.0000E+00	-1.5266E+01	-1.8500E+00	1.0145E+00	-2.4017E+00	-1.1873E+01	6.5714E+00	-3.6257E+00
2.30	1.0000E-01	-1.3000E+00	1.0000E+00	-1.5801E+01	-2.1668E+00	8.2404E-01	-3.1789E+00	-9.3349E+00	6.0914E+00	-3.9882E+00
2.35	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6216E+01	-2.4586E+00	6.1636E-01	-3.9798E+00	-7.0352E+00	5.5794E+00	-4.3123E+00
2.40	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6503E+01	-2.7242E+00	3.9348E-01	-4.7984E+00	-4.3991E+00	5.0401E+00	-4.5961E+00
2.45	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6654E+01	-2.9623E+00	1.5746E-01	-5.6279E+00	-1.6518E+00	4.4782E+00	-4.8378E+00
2.50	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6666E+01	-3.1718E+00	-8.9578E-02	-6.4615E+00	1.1810E+00	3.8985E+00	-5.0363E+00
2.55	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6535E+01	-3.3519E+00	-7.2921E+00	4.0733E+00	3.3059E+00	-5.1905E+00	
2.60	1.0000E-01	-1.3000E+00	1.0000E+00	-1.6259E+01	-3.5022E+00	-6.0788E-01	-8.1126E+00	6.9990E+00	2.7055E+00	-5.2999E+00
2.65	1.0000E-01	-1.3000E+00	1.0000E+00	-1.5835E+01	-3.6224E+00	-8.7467E-01	-8.9155E+00	9.9321E+00	2.1020E+00	-5.3643E+00
2.70	1.0000E-01	-1.3000E+00	1.0000E+00	-1.5266E+01	-3.7125E+00	-1.1436E+00	-9.6937E+00	1.2847E+01	1.5005E+00	-5.3838E+00
2.75	1.0000E-01	-1.3000E+00	1.0000E+00	-1.4551E+01	-3.7726E+00	-1.4123E+00	-1.0440E+01	1.5719E+01	9.0576E-01	-5.3588E+00
2.80	1.0000E-01	-1.3000E+00	1.0000E+00	-1.3695E+01	-3.8032E+00	-1.6787E+00	-1.1146E+01	1.8524E+01	3.2238E-01	-5.2903E+00
2.85	1.0000E-01	-1.3000E+00	1.0000E+00	-1.2700E+01	-3.8051E+00	-1.9406E+00	-1.1807E+01	2.1237E+01	-2.4506E-01	-5.1792E+00
2.90	1.0000E-01	-1.3000E+00	1.0000E+00	-1.1573E+01	-3.7791E+00	-2.1960E+00	-1.2414E+01	2.3837E+01	-7.9224E-01	-5.0271E+00
2.95	1.0000E-01	-1.3000E+00	1.0000E+00	-1.0319E+01	-3.7263E+00	-2.4427E+00	-1.2962E+01	2.6303E+01	-1.3150E+00	-4.8358E+00
3.00	1.0000E-01	-1.3000E+00	1.0000E+00	-8.9453E+00	-3.6480E+00	-2.6789E+00	-1.3444E+01	2.8615E+01	-1.8096E+00	-4.6071E+00
3.05	1.0000E-01	-1.3000E+00	1.0000E+00	-7.4603E+00	-3.5458E+00	-2.9028E+00	-1.3855E+01	3.0755E+01	-2.2723E+00	-4.3436E+00
3.10	1.0000E-01	-1.3000E+00	1.0000E+00	-5.8730E+00	-3.4214E+00	-3.1127E+00	-1.4188E+01	3.2705E+01	-2.6998E+00	-4.0476E+00
3.15	1.0000E-01	-1.3000E+00	1.0000E+00	-4.1932E+00	-3.2765E+00	-3.3071E+00	-1.4441E+01	3.4451E+01	-3.0892E+00	-3.7220E+00
3.20	1.0000E-01	-1.3000E+00	1.0000E+00	-2.4315E+00	-3.1131E+00	-3.4845E+00	-1.4606E+01	3.5981E+01	-3.4379E+00	-3.3696E+00
3.25	1.0000E-01	-1.3000E+00	1.0000E+00	-5.9894E-01	-2.9334E+00	-3.6437E+00	-1.4682E+01	3.7282E+01	-3.7435E+00	-2.9937E+00
3.30	1.0000E-01	-1.3000E+00	1.0000E+00	1.2928E+00	-2.7395E+00	-3.7835E+00	-1.4665E+01	3.8346E+01	-4.0041E+00	-2.5974E+00
3.35	1.0000E-01	-1.3000E+00	1.0000E+00	3.2316E+00	-2.5338E+00	-3.9031E+00	-1.4552E+01	3.9165E+01	-4.2183E+00	-2.1841E+00

APPENDIX B – Continued

3.40	1.0000E-01	-1.3000E+00	1.0000E+00	5.2051E+00	2.3185E+00	-4.0017E+00	-1.4342E+01	3.9735E+01	-4.3849E+00	-1.7574E+00
3.45	1.0000E-01	-1.3000E+00	1.0000E+00	7.2009E+00	2.0961E+00	-4.0787E+00	-1.4032E+01	4.0052E+01	-4.5030E+00	-1.3206E+00
3.50	1.0000E-01	-1.3000E+00	1.0000E+00	9.2061E+00	1.8690E+00	-4.1337E+00	-1.3621E+01	4.0115E+01	-4.5724E+00	-8.7744E-01
3.55	1.0000E-01	-1.3000E+00	1.0000E+00	1.1208E+01	1.6397E+00	-4.1664E+00	-1.3111E+01	3.9925E+01	-4.5931E+00	-4.3136E-01
3.60	1.0000E-01	-1.3000E+00	1.0000E+00	1.3194E+01	1.4105E+00	-4.1768E+00	-1.2501E+01	3.9485E+01	-4.5654E+00	1.4084E-02
3.65	1.0000E-01	-1.3000E+00	1.0000E+00	1.5153E+01	1.1839E+00	-4.1650E+00	-1.1792E+01	3.8800E+01	-4.4901E+00	4.5541E-01
3.70	1.0000E-01	-1.3000E+00	1.0000E+00	1.7071E+01	9.6227E+00	-4.1314E+00	-1.0986E+01	3.7877E+01	-4.3685E+00	8.8923E-01
3.75	1.0000E-01	-1.3000E+00	1.0000E+00	1.8936E+01	7.4783E+00	-4.0763E+00	-1.0086E+01	3.6725E+01	-4.2018E+00	1.3122E+00
3.80	1.0000E-01	-1.3000E+00	1.0000E+00	2.0739E+01	5.4281E+00	-4.0004E+00	-9.0936E+00	3.5935E+01	-3.9921E+00	1.7213E+00
3.85	1.0000E-01	-1.3000E+00	1.0000E+00	2.2468E+01	3.4931E+00	-3.9045E+00	-8.0131E+00	3.3773E+01	-3.7414E+00	2.1133E+00
3.90	1.0000E-01	-1.3000E+00	1.0000E+00	2.4113E+01	1.6932E+00	-3.7894E+00	-6.8482E+00	3.1999E+01	-3.4521E+00	2.4855E+00
3.95	1.0000E-01	-1.3000E+00	1.0000E+00	2.5665E+01	4.6996E-03	-3.6563E+00	-5.6033E+00	3.0046E+01	-3.1269E+00	2.8352E+00
4.00	1.0000E-01	-1.3000E+00	1.0000E+00	2.7115E+01	-1.4282E+01	-3.5063E+00	-4.2834E+00	2.7930E+01	-2.7688E+00	3.1600E+00
4.05	1.0000E-01	-1.3000E+00	1.0000E+00	2.8456E+01	-2.7168E+01	-3.3407E+00	-2.8936E+00	2.5667E+01	-2.3809E+00	3.4576E+00
4.10	1.0000E-01	-1.3000E+00	1.0000E+00	2.9680E+01	-3.8048E+01	-3.1610E+00	-1.4398E+00	2.3276E+01	-1.9667E+00	3.7261E+00
4.15	1.0000E-01	-1.3000E+00	1.0000E+00	3.0781E+01	-4.6797E+01	-2.9686E+00	7.2295E-02	2.0775E+01	-1.5296E+00	3.9638E+00
4.20	1.0000E-01	-1.3000E+00	1.0000E+00	3.1756E+01	-5.3311E+01	-2.7652E+00	1.6363E+00	1.8185E+01	-1.0732E+00	4.1691E+00
4.25	1.0000E-01	-1.3000E+00	1.0000E+00	3.2599E+01	-5.7504E+01	-2.5523E+00	3.2457E+00	1.5524E+01	-6.0139E+01	4.3409E+00
4.30	1.0000E-01	-1.3000E+00	1.0000E+00	3.3307E+01	-5.9306E+01	-2.3317E+00	4.8939E+00	1.2814E+01	-1.1794E+01	4.4783E+00
4.35	1.0000E-01	-1.3000E+00	1.0000E+00	3.3880E+01	-5.8670E+01	-2.1050E+00	6.5741E+00	1.0074E+01	3.7327E+01	4.5807E+00
4.40	1.0000E-01	-1.3000E+00	1.0000E+00	3.4315E+01	-5.5567E+01	-1.8742E+00	8.2796E+00	7.3245E+00	8.6835E+01	4.6476E+00
4.45	1.0000E-01	-1.3000E+00	1.0000E+00	3.4612E+01	-4.9986E+01	-1.6409E+00	1.0003E+01	4.5866E+00	1.3634E+00	4.6790E+00
4.50	1.0000E-01	-1.3000E+00	1.0000E+00	3.4774E+01	-4.1939E+01	-1.4069E+00	1.1739E+01	1.8798E+00	1.8547E+00	4.6751E+00
4.55	1.0000E-01	-1.3000E+00	1.0000E+00	3.4801E+01	-3.1452E+01	-1.1739E+00	1.3478E+01	7.7636E-01	2.3384E+00	4.6365E+00
4.60	1.0000E-01	-1.3000E+00	1.0000E+00	3.4697E+01	-1.8574E+01	-9.4379E+01	1.5216E+01	3.3638E+00	2.8109E+00	4.5637E+00
4.65	1.0000E-01	-1.3000E+00	1.0000E+00	3.4466E+01	-3.3681E+02	-7.1811E+01	1.6946E+01	5.8637E+00	3.2686E+00	4.4579E+00
4.70	1.0000E-01	-1.3000E+00	1.0000E+00	3.4113E+01	-1.4083E+01	-4.9853E+01	1.8661E+01	8.2591E+00	3.7084E+00	4.3202E+00
4.75	1.0000E-01	-1.3000E+00	1.0000E+00	3.3642E+01	-3.3680E+01	-2.8660E+01	2.0355E+01	-1.0534E+01	4.1269E+00	4.1521E+00
4.80	1.0000E-01	-1.3000E+00	1.0000E+00	3.3061E+01	-5.5311E+01	-8.3799E+02	2.2023E+01	-1.2673E+01	4.5212E+00	3.9553E+00
4.85	1.0000E-01	-1.3000E+00	1.0000E+00	3.2377E+01	-7.8847E+01	-1.0849E+01	2.3660E+01	-1.4666E+01	4.8886E+00	3.7317E+00
4.90	1.0000E-01	-1.3000E+00	1.0000E+00	3.1598E+01	1.0415E+00	2.8896E+01	2.5260E+01	-1.6491E+01	5.2265E+00	3.4834E+00
4.95	1.0000E-01	-1.3000E+00	1.0000E+00	3.0731E+01	1.3106E+00	4.5645E+01	2.6818E+01	-1.8146E+01	5.5328E+00	3.2126E+00
5.00	-2.3500E+00	-1.3000E+00	1.0000E+00	2.9287E+01	1.6636E+00	6.0711E+01	2.8319E+01	-3.9593E+01	8.5789E+00	2.8107E+00
5.05	-4.8000E+00	-1.3000E+00	1.0000E+00	2.6784E+01	2.1667E+00	7.3170E+01	2.9721E+01	-6.0442E+01	1.1536E+01	2.1712E+00
5.10	-7.2500E+00	-1.3000E+00	1.0000E+00	2.3257E+01	2.8150E+00	8.1853E+01	3.0972E+01	-8.0572E+01	1.4379E+01	1.3015E+00
5.15	-9.7000E+00	-1.3000E+00	1.0000E+00	1.8743E+01	3.6021E+00	8.5630E+01	3.2022E+01	-9.9863E+01	1.7086E+01	2.1072E-01
5.20	-9.5000E+00	-9.0000E+00	1.0000E+00	1.3870E+01	4.4083E+00	8.3825E+01	3.2837E+01	-9.4955E+01	1.5140E+01	-9.2989E-01
5.25	-9.3000E+00	-5.0000E+00	1.0000E+00	9.2564E+00	5.1114E+00	7.6614E+01	3.3414E+01	-8.9510E+01	1.3098E+01	-1.9509E+00
5.30	-7.9000E+00	-5.0000E+00	1.0000E+00	5.1301E+00	5.7202E+00	6.4643E+01	3.3773E+01	-7.5464E+01	1.1110E+01	-2.8329E+00
5.35	-6.5000E+00	-5.0000E+00	1.0000E+00	1.7129E+00	6.2247E+00	4.8654E+01	3.3944E+01	-6.1161E+01	9.0562E+00	-3.5572E+00
5.40	-5.1000E+00	-5.0000E+00	1.0000E+00	-9.8466E+00	6.6252E+00	2.9447E+01	3.3962E+01	-4.6694E+01	6.9557E+00	-4.1197E+00
5.45	-3.7000E+00	-5.0000E+00	1.0000E+00	-2.9568E+00	6.9199E+00	7.8374E+02	3.3862E+01	-3.2160E+01	4.8260E+00	-4.5179E+00
5.50	-2.3000E+00	-5.0000E+00	1.0000E+00	-4.2024E+00	7.1078E+00	-1.5349E+01	3.3683E+01	-1.7651E+01	2.6852E+00	-4.7503E+00
5.55	-9.0000E+00	-5.0000E+00	1.0000E+00	-4.7252E+00	7.1887E+00	-3.9283E+01	3.3459E+01	-3.2614E+01	5.5144E+00	-4.8169E+00
5.60	5.0000E+00	-5.0000E+00	1.0000E+00	-4.5334E+00	7.1635E+00	-6.3137E+01	3.3227E+01	-1.0916E+01	-1.5576E+00	-4.7189E+00
5.65	1.9000E+00	-5.0000E+00	1.0000E+00	-3.6399E+00	7.0338E+00	-8.6095E+01	3.3022E+01	-2.4793E+01	-3.6245E+00	-4.4589E+00
5.70	3.3000E+00	-5.0000E+00	1.0000E+00	-2.0619E+00	6.8021E+00	-1.0736E+00	3.2879E+01	-3.8283E+01	-5.6324E+00	-4.0406E+00
5.75	4.7000E+00	-5.0000E+00	1.0000E+00	1.7912E+00	6.4719E+00	-1.2614E+00	3.2832E+01	-5.1302E+01	-7.5652E+00	-3.4690E+00
5.80	6.1000E+00	-5.0000E+00	1.0000E+00	3.0577E+00	6.0473E+00	-1.4169E+00	3.2913E+01	-6.3772E+01	-9.4075E+00	-2.7501E+00
5.85	7.5000E+00	-5.0000E+00	1.0000E+00	6.5446E+00	5.5331E+00	-1.5330E+00	3.3153E+01	-7.5620E+01	-1.1145E+01	-1.8910E+00
5.90	8.9000E+00	-5.0000E+00	1.0000E+00	1.0607E+00	4.9349E+00	-1.6028E+00	3.3581E+01	-8.5777E+01	-1.2763E+01	-8.9988E+00
5.95	1.0300E+01	-5.0000E+00	1.0000E+00	1.5228E+01	4.2591E+00	-1.6199E+00	3.4227E+01	-9.7181E+01	-1.4250E+01	2.1420E-01
6.00	1.0400E+01	-5.0000E+00	1.0000E+00	2.0045E+01	3.5492E+00	-1.5799E+00	3.5109E+01	-9.6178E+01	-1.4124E+01	1.3825E+00
6.05	1.0500E+01	-5.0000E+00	1.0000E+00	2.4815E+01	2.8488E+00	-1.4818E+00	3.6231E+01	-9.4531E+01	-1.3873E+01	2.5364E+00
6.10	1.0600E+01	-5.0000E+00	1.0000E+00	2.9488E+01	2.1640E+00	-1.3265E+00	3.7589E+01	-9.2260E+01	-1.3501E+01	3.6673E+00
6.15	1.0700E+01	-5.0000E+00	1.0000E+00	3.4031E+01	1.5057E+00	-1.1155E+00	3.9178E+01	-8.9390E+01	-1.3013E+01	4.7667E+00
6.20	9.5000E+00	-5.0000E+00	1.0000E+00	3.8152E+01	9.0134E+01	-8.5194E+01	4.0983E+01	-7.5351E+01	-1.0942E+01	5.7674E+00
6.25	8.3000E+00	-5.0000E+00	1.0000E+00	4.1562E+01	4.0757E+01	-5.4241E+01	4.2977E+01	-6.0989E+01	-8.7948E+00	6.6041E+00
6.30	7.1000E+00	-5.0000E+00	1.0000E+00	4.4248E+01	2.2710E+02	-1.9526E+01	4.5123E+01	-6.4603E+01	-6.5891E+00	7.2719E+00
6.35	5.9000E+00	-5.0000E+00	1.0000E+00	4.6202E+01	-2.5081E+01	1.8099E+01	4.7385E+01	-3.1694E+01	-4.3446E+00	7.7675E+00
6.40	4.7000E+00	-5.0000E+00	1.0000E+00	4.7418E+01	-4.1152E+01	5.7768E+01	4.9727E+01	-1.6960E+01	-2.0797E+00	8.0890E+00
6.45	3.5000E+00	-5.0000E+00	1.0000E+00	4.7900E+01	-4.5890E+01	9.8670E+01	5.2111E+01	-2.3014E+01	-1.8572E+01	8.2359E+00
6.50	2.3000E+00	-5.0000E+00	1.0000E+00	4.7653E+01	-3.9337E+01	1.3975E+00	5.4501E+01	-1.2184E+01	2.4332E+00	8.2088E+00

6.55	1.1000E+00	-5.0000E-01	1.0000E+00	4.6687E+01	-2.1630E-01	1.8032E+00	5.6860E+01	-2.6402E+01	4.6443E+00	8.0100E+00
6.60	-1.0000E-01	-5.0000E-01	1.0000E+00	4.5020E+01	7.0027E-02	2.1947E+00	5.9154E+01	-4.0259E+01	6.8009E+00	7.6429E+00
6.65	-1.3000E+00	-5.0000E-01	1.0000E+00	4.2670E+01	4.6247E-01	2.5638E+00	6.1347E+01	-5.3666E+01	8.8859E+00	7.1122E+00
6.70	-2.5000E+00	-5.0000E-01	1.0000E+00	3.9663E+01	9.5702E-01	2.9024E+00	6.3406E+01	-6.6539E+01	1.0882E+01	6.4239E+00
6.75	-3.7000E+00	-5.0000E-01	1.0000E+00	3.6027E+01	1.5489E+00	3.2028E+00	6.5298E+01	-7.8798E+01	1.2775E+01	5.5851E+00
6.80	-3.7000E+00	9.0000E-01	1.0000E+00	3.2185E+01	2.0676E+00	3.4626E+00	6.7004E+01	-7.4826E+01	7.9599E+00	4.8002E+00
6.85	-3.7000E+00	2.3000E+00	1.0000E+00	2.8549E+01	2.3450E+00	3.6894E+00	6.8523E+01	-7.0551E+01	3.1238E+00	4.2694E+00
6.90	-3.7000E+00	3.7000E+00	1.0000E+00	2.5132E+01	2.3806E+00	3.8961E+00	6.9865E+01	-6.6120E+01	-1.7056E+00	3.9910E+00
6.95	-3.7000E+00	5.1000E+00	1.0000E+00	2.1937E+01	2.1754E+00	4.0951E+00	7.1042E+01	-6.1678E+01	-6.5013E+00	3.9611E+00
7.00	-3.7000E+00	5.0900E+00	1.0000E+00	1.8816E+01	1.8637E+00	4.2951E+00	7.2061E+01	-6.3164E+01	-5.9689E+00	4.0313E+00
7.05	-3.7000E+00	5.0800E+00	1.0000E+00	1.5620E+01	1.5785E+00	4.4974E+00	7.2922E+01	-6.4656E+01	-5.4370E+00	4.0554E+00
7.10	-3.7000E+00	5.0700E+00	1.0000E+00	1.2350E+01	1.3199E+00	4.6938E+00	7.3622E+01	-6.6125E+01	-4.9105E+00	4.0342E+00
7.15	-3.7000E+00	5.0600E+00	1.0000E+00	9.0083E+00	1.0873E+00	4.9001E+00	7.4156E+01	-6.7549E+01	-4.3944E+00	3.9684E+00
7.20	-3.7000E+00	5.0500E+00	1.0000E+00	5.5966E+00	8.8018E-01	5.0960E+00	7.4521E+01	-6.8902E+01	-3.8931E+00	3.8955E+00
7.25	-3.7000E+00	5.0400E+00	1.0000E+00	2.1196E+00	6.9766E-01	5.2853E+00	7.4715E+01	-7.0162E+01	-3.4111E+00	3.7091E+00
7.30	-3.7000E+00	5.0300E+00	1.0000E+00	-1.4176E+00	5.3868E-01	5.4662E+00	7.4732E+01	-7.1306E+01	-2.9525E+00	3.5189E+00
7.35	-3.7000E+00	5.0200E+00	1.0000E+00	-5.0088E+00	4.0195E-01	5.6366E+00	7.4572E+01	-7.2316E+01	-2.5214E+00	3.2912E+00
7.40	-3.7000E+00	5.0100E+00	1.0000E+00	-8.6466E+00	2.8602E-01	5.7947E+00	7.4231E+01	-7.3171E+01	-2.1213E+00	3.0285E+00
7.45	-3.7000E+00	5.0000E+00	1.0000E+00	-1.2323E+01	1.8925E-01	5.9389E+00	7.3707E+01	-7.3855E+01	-1.7555E+00	2.7334E+00
7.50	-3.7000E+00	4.9900E+00	1.0000E+00	-1.6029E+01	1.0985E-01	6.0675E+00	7.2998E+01	-7.4353E+01	-1.4270E+00	2.4098E+00
7.55	-3.7000E+00	4.9800E+00	1.0000E+00	-1.9755E+01	4.5884E-02	6.1793E+00	7.2103E+01	-7.4652E+01	-1.1384E+00	2.0977E+00
7.60	-3.7000E+00	4.9700E+00	1.0000E+00	-2.3491E+01	-4.6973E-03	6.2729E+00	7.1022E+01	-7.4741E+01	-8.9200E-01	1.5835E+00
7.65	-3.7000E+00	4.9600E+00	1.0000E+00	-2.7225E+01	-4.4054E-02	6.3473E+00	6.9754E+01	-7.4610E+01	-6.8966E-01	1.2894E+00
7.70	-3.7000E+00	4.9500E+00	1.0000E+00	-3.0948E+01	-7.4426E-02	6.4015E+00	6.8300E+01	-7.4253E+01	-5.3285E-01	8.7894E-01
7.75	-3.7000E+00	4.9400E+00	1.0000E+00	-3.4644E+01	-9.8120E-02	6.4530E+00	6.6660E+01	-7.3664E+01	-4.2266E-01	4.5557E-01
7.80	-3.7000E+00	4.9300E+00	1.0000E+00	-3.8310E+01	-1.1748E-01	6.4470E+00	6.4836E+01	-7.2840E+01	-3.5978E-01	2.3321E-02
7.85	-3.7000E+00	4.9200E+00	1.0000E+00	-4.1927E+01	-1.3489E-01	6.4372E+00	6.2830E+01	-7.1780E+01	-3.4449E-01	-4.1464E-01
7.90	-3.7000E+00	4.9100E+00	1.0000E+00	-4.5485E+01	-1.5273E-01	6.4054E+00	6.0644E+01	-7.0487E+01	-3.7670E-01	-8.5452E-01
7.95	-3.7000E+00	4.9000E+00	1.0000E+00	-4.8972E+01	-1.7335E-01	6.3517E+00	5.8282E+01	-6.8962E+01	-4.5590E-01	-1.2928E+00
8.00	-3.7000E+00	3.4000E+00	1.0000E+00	-5.2531E+01	-5.9847E-02	6.2724E+00	5.5744E+01	-7.3342E+01	4.9858E+00	-1.8769E+00
8.05	-3.7000E+00	1.9000E+00	1.0000E+00	-5.6297E+01	3.2248E-01	6.1567E+00	5.3023E+01	-7.7232E+01	1.0292E+01	-2.7501E+00
8.10	-3.7000E+00	4.0000E-01	1.0000E+00	-6.0242E+01	9.6614E-01	5.9903E+00	5.0109E+01	-8.0479E+01	1.5434E+01	-3.8987E+00
8.15	-3.7000E+00	-1.1000E+00	1.0000E+00	-6.4331E+01	1.8622E+00	5.7599E+00	4.6994E+01	-8.2942E+01	2.0384E+01	-5.3073E+00
8.20	-3.4429E+00	-1.0357E+00	1.0000E+00	-6.8307E+01	2.8470E+00	5.4572E+00	4.3677E+01	-7.5957E+01	1.8982E+01	-6.7885E+00
8.25	-3.1857E+00	-9.7143E-01	1.0000E+00	-7.1915E+01	3.7582E+00	5.0832E+00	4.0170E+01	-6.8264E+01	1.7441E+01	-8.1580E+00
8.30	-2.9286E+00	-9.0714E-01	1.0000E+00	-7.5123E+01	4.5891E+00	4.6437E+00	3.6493E+01	-5.9940E+01	1.5775E+01	-9.4069E+00
8.35	-2.6714E+00	-8.4286E-01	1.0000E+00	-7.9700E+01	5.3339E+00	4.1450E+00	3.2666E+01	-5.1063E+01	1.3999E+01	-1.0527E+01
8.40	-2.4143E+00	-7.7857E-01	1.0000E+00	-8.0222E+01	5.9875E+00	3.5936E+00	2.8712E+01	-4.1717E+01	1.2128E+01	-1.1512E+01
8.45	-2.1571E+00	-7.1429E-01	1.0000E+00	-8.2066E+01	6.5455E+00	2.9965E+00	2.4653E+01	-3.1985E+01	1.0179E+01	-1.2355E+01
8.50	-1.9000E+00	-6.5000E-01	1.0000E+00	-8.3415E+01	7.0044E+00	2.3608E+00	2.0514E+01	-2.1957E+01	8.1672E+00	-1.3052E+01
8.55	-1.6429E+00	-5.8571E-01	1.0000E+00	-8.4258E+01	7.3615E+00	1.6940E+00	1.6321E+01	-1.1720E+01	6.1097E+00	-1.3600E+01
8.60	-1.3857E+00	-5.2143E-01	1.0000E+00	-8.4586E+01	7.6149E+00	1.0036E+00	1.2098E+01	-1.3598E+00	4.0232E+00	-1.3996E+01
8.65	-1.1286E+00	-4.5714E-01	1.0000E+00	-8.4394E+01	7.7636E+00	2.9723E+00	7.8721E+00	9.0330E+00	1.9244E+00	-1.4240E+01
8.70	-8.7143E-01	-3.9286E-01	1.0000E+00	-8.3683E+01	7.8075E+00	4.1755E+00	3.6685E+00	1.9373E+00	-1.6998E-01	-1.4332E+01
8.75	-6.1429E-01	-3.2857E-01	1.0000E+00	-8.2459E+01	7.7470E+00	-1.1331E+00	-4.8669E-01	2.9575E+01	-2.2437E+00	-1.4272E+01
8.80	-3.5714E-01	-2.6429E-01	1.0000E+00	-8.0730E+01	7.5837E+00	-1.8420E+00	-4.5680E+00	3.9555E+01	-4.2809E+00	-1.4064E+01
8.85	-1.0000E-01	-2.0000E-01	1.0000E+00	-7.8509E+01	7.3198E+00	-2.5368E+00	-8.5505E+00	4.9234E+01	-6.2660E+00	-1.3711E+01
8.90	-9.5455E-02	-1.9091E-01	1.0000E+00	-7.5870E+01	6.9706E+00	-3.2109E+00	-1.2411E+01	5.6250E+01	-7.6920E+00	-1.3235E+01
8.95	-9.0909E-02	-1.8182E-01	1.0000E+00	-7.2891E+01	6.5519E+00	-3.8586E+00	-1.6132E+01	6.2867E+01	-9.0460E+00	-1.2659E+01
9.00	-8.6364E-02	-1.7273E-01	1.0000E+00	-6.9591E+01	6.0674E+00	-4.4752E+00	-1.9695E+01	6.9037E+01	-1.0318E+01	-1.1987E+01
9.05	-8.1818E-02	-1.6364E-01	1.0000E+00	-6.5959E+01	5.5216E+00	-5.0559E+00	-2.3086E+01	7.4713E+01	-1.1499E+01	-1.1228E+01
9.10	-7.7273E-02	-1.5455E-01	1.0000E+00	-6.2129E+01	4.9191E+00	-5.5966E+00	-2.6290E+01	7.9855E+01	-1.2582E+01	-1.0387E+01
9.15	-7.2727E-02	-1.4545E-01	1.0000E+00	-5.8019E+01	4.2652E+00	-6.0934E+00	-2.9295E+01	8.4427E+01	-1.3558E+01	-9.4726E+00
9.20	-6.8182E-02	-1.3636E-01	1.0000E+00	-5.3696E+01	3.5652E+00	-6.5427E+00	-3.2088E+01	8.8397E+01	-1.4421E+01	-8.4927E+00
9.25	-6.3636E-02	-1.2727E-01	1.0000E+00	-4.9190E+01	2.8251E+00	-6.9417E+00	-3.4661E+01	9.1739E+01	-1.5166E+01	-7.4559E+00
9.30	-5.9091E-02	-1.1818E-01	1.0000E+00	-4.4533E+01	2.0507E+00	-7.2875E+00	-3.7005E+01	9.4432E+01	-1.5789E+01	-6.3710E+00

APPENDIX B – Continued

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9.35 -5.4545E-02 -1.0909E-01 1.0000E+00 -3.9758E+01 1.2483E+00 -7.5781E+00 -3.9113E+01 9.6462E+01 -1.6285E+01 -5.2471E+00
9.40 -5.0000E-02 -1.0000E-01 1.0000E+00 -3.4898E+01 4.2431E-01 -7.8117E+00 -4.0979E+01 9.7817E+01 -1.6654E+01 -4.0935E+00
9.45 -4.5455E-02 -9.0909E-02 1.0000E+00 -2.9988E+01 -4.1489E-01 -7.9871E+00 -4.2602E+01 9.8493E+01 -1.6893E+01 -2.9195E+00
9.50 -4.0909E-02 -8.1818E-02 1.0000E+00 -2.5060E+01 -1.2628E+00 -8.1034E+00 -4.3978E+01 9.8492E+01 -1.7002E+01 -1.7345E+00
9.55 -3.6364E-02 -7.2727E-02 1.0000E+00 -2.0150E+01 -2.1129E+00 -8.1605E+00 -4.5108E+01 9.7819E+01 -1.6981E+01 -5.4806E-01
9.60 -3.1818E-02 -6.3636E-02 1.0000E+00 -1.5289E+01 -2.9588E+00 -8.1584E+00 -4.5993E+01 9.6485E+01 -1.6834E+01 6.3068E-01
9.65 -2.7273E-02 -5.4545E-02 1.0000E+00 -1.0512E+01 -3.7942E+00 -8.0977E+00 -4.6638E+01 9.4507E+01 -1.6561E+01 1.7926E+00
9.70 -2.2727E-02 -4.5455E-02 1.0000E+00 -5.8489E+00 -4.6129E+00 -7.9795E+00 -4.7047E+01 9.1906E+01 -1.6168E+01 2.9289E+00
9.75 -1.8182E-02 -3.6364E-02 1.0000E+00 -1.3311E+00 -5.4090E+00 -7.8053E+00 -4.7225E+01 8.8708E+01 -1.5657E+01 4.0310E+00
9.80 -1.3636E-02 -2.7273E-02 1.0000E+00 3.0125E+00 -6.1768E+00 -7.5771E+00 -4.7182E+01 8.4943E+01 -1.5036E+01 5.0908E+00
9.85 -9.0909E-03 -1.8182E-02 1.0000E+00 7.1543E+00 -6.9108E+00 -7.2971E+00 -4.6927E+01 8.0645E+01 -1.4310E+01 6.1005E+00
9.90 -4.5455E-03 -9.0909E-03 1.0000E+00 1.1069E+01 -7.6061E+00 -6.9680E+00 -4.6471E+01 7.5853E+01 -1.3485E+01 7.0531E+00
9.95 -5.6932E-13 -1.1386E-12 1.0000E+00 1.4732E+01 -8.2578E+00 -6.5928E+00 -4.5825E+01 7.0606E+01 -1.2570E+01 7.9417E+00

THE VALUE OF THE FIT ERROR IS 2.08800E-05 FOR THE ITERATION NUMBER 9

MATRIX EE*D HAS 1 ROWS AND 7 COLUMNS
1.4898E-06 3.8844E-08 1.4690E-06 1.7081E-05 5.6953E-07
4.7047E-08 1.8428E-07

MATRIX RMS HAS 1 ROWS AND 7 COLUMNS
1.2206E-03 1.9709E-04 1.2120E-03 4.1330E-03 7.5467E-04
2.1690E-04 4.2928E-04

MATRIX GRAD HAS 1 ROWS AND 16 COLUMNS
9.1397E-05 -9.2442E-06 1.4474E-C5 -1.9878E-05 9.3030E-06
-9.6792E-06 4.4500E-04 9.2840E-05 -5.0273E-05 7.7592E-05
-1.2587E-05 4.5126E-05 -1.4111E-03 -7.6391E-07 3.0171E-07
2.0880E-05

WOULD YOU BELIEVE.....
8.3496613E-05

MATRIX DC HAS 1 ROWS AND 15 COLUMNS
-1.9149E-06 -1.8729E-06 -1.7436E-C5 9.9519E-08 -7.6805E-06
-1.2673E-04 4.9877E-07 2.9333E-06 4.5277E-05 6.6766E-07
2.6512E-06 1.8164E-05 -3.6874E-C7 1.8826E-06 -1.1702E-05

MATRIX A HAS 4 ROWS AND 4 COLUMNS
-2.7396E-01 6.3028E-02 -1.1400E+C1 -0.
1.3100E-03 -2.0814E-01 2.1701E+00 -0.
8.2997E-02 -1.0000E+00 -3.1494E-01 7.8900E-02
1.0000E+00 -0. -0. -0.

MATRIX B HAS 4 ROWS AND 3 COLUMNS
8.2232E+00 4.1772E+00 1.9426E-C3
-1.1406E+00 -3.7614E+00 -5.0107E-04
-0. -0. 3.5644E-C4
-0. -0. -0.

MATRIX DZ1 HAS 1 ROWS AND 7 COLUMNS
0. 0. 0. 0.
0. 0. 0.

MATRIX DZN HAS 1 ROWS AND 7 COLUMNS
0. 0. 0. 0.
0. 0. 0.

MATRIX F HAS 3 ROWS AND 4 COLUMNS
-2.7396E-01 6.3028E-02 -1.1400E+C1 0.
1.3100E-03 -2.0814E-01 2.1701E+00 0.
8.2997E-02 -1.0000E+00 -3.1494E-01 7.8900E-02

MATRIX G HAS 3 ROWS AND 3 COLUMNS
8.2232E+00 4.1772E+00 1.9426E-C3
-1.1406E+00 -3.7614E+00 -5.0107E-C4
0. 0. 3.5644E-04

8.3496613E-05

MATRIX S-1S HAS 15 ROWS AND 15 COLUMNS
1.0000E+00 -1.7053E-13 -5.3814E-15 8.8818E-15 1.0214E-14
-1.2790E-13 -3.7517E-12 -1.0658E-12 3.6948E-13 -2.4025E-13
3.0909E-13 -7.9581E-13 2.4727E-12 1.0658E-14 1.9185E-13
-2.7569E-12 1.0000E+00 -2.9843E-13 8.1712E-14 3.3751E-14

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-6.8212E-13 -2.2965E-11 -2.7853E-12 1.5632E-13 -3.1442E-13
-4.2633E-14 -2.6716E-12 -4.5475E-12 4.6754E-12 5.4001E-13
-7.8444E-12 -1.1795E-12 1.0000E+00 7.9936E-13 -4.5475E-13
-1.1227E-12 -7.3214E-11 -4.3201E-12 3.8369E-12 -3.9933E-12
-3.9790E-13 -4.8885E-12 4.5020E-11 1.1966E-11 1.7621E-12
5.2225E-13 -3.0287E-13 -8.7486E-14 1.0000E+00 2.3759E-14
-9.2371E-14 -6.7359E-12 -1.6698E-13 -3.8192E-13 3.3840E-13
-2.5668E-13 -6.5015E-13 -8.4270E-12 1.3767E-12 1.7231E-13
-6.4801E-12 -8.2423E-13 -8.4377E-14 1.2790E-13 1.0000E+00
-4.5475E-13 -1.6371E-11 -4.0927E-12 1.3927E-12 -1.1280E-12
1.5632E-12 -3.2969E-12 1.6598E-11 5.6843E-14 7.3896E-13
-3.5016E-11 -5.9117E-12 5.7909E-13 3.6948E-13 8.5265E-13
1.0000E+00 -1.4552E-10 -3.2742E-11 8.1855E-12 -2.4869E-12
9.3223E-12 -2.4556E-11 7.8217E-11 2.2737E-13 1.0914E-11
3.0020E-13 3.0198E-14 -4.4409E-16 -5.5511E-15 -7.7716E-16
2.9310E-14 1.0000E+00 2.2737E-13 -7.1942E-14 5.5844E-14
-8.0824E-14 1.3856E-13 -1.0587E-12 3.1086E-14 -4.5185E-14
1.2506E-12 2.0606E-13 9.1926E-14 -4.7962E-14 2.6645E-15
2.1316E-13 8.4128E-12 1.0000E+00 -3.3396E-13 3.8947E-13
-3.0553E-13 1.0232E-12 -4.7746E-12 -6.1817E-13 -2.9843E-13
1.7906E-12 2.6290E-13 1.8630E-13 -8.1712E-14 2.1316E-14
2.4869E-13 9.7771E-12 1.5916E-12 1.0000E+00 3.9702E-13
-3.4106E-13 1.4495E-12 -1.2619E-11 -9.1660E-13 -3.8369E-13
1.0658E-13 -2.1316E-14 3.9302E-14 -2.5757E-14 1.7764E-14
1.7764E-14 3.4106E-13 1.2790E-13 5.3291E-14 1.0000E+00
6.7502E-14 4.2633E-14 -9.6634E-13 -8.5265E-14 -7.1054E-15
-5.5778E-13 -2.6956E-13 5.4339E-13 -2.6268E-13 2.1338E-13
-7.7716E-14 5.8265E-12 -8.5620E-13 -3.7303E-13 8.9195E-13
1.0000E+00 -3.7659E-13 -1.7046E-11 -1.2186E-12 1.0125E-13
4.4338E-12 1.2150E-12 -1.7653E-13 -7.9936E-14 -1.8652E-13
1.0445E-12 2.6375E-11 7.8444E-12 -1.0232E-12 9.8588E-14
-1.8829E-12 1.0000E+00 -6.3665E-12 -3.8369E-13 -1.6485E-12
-3.1086E-14 -1.0658E-14 -2.6062E-14 1.2157E-14 -6.4393E-15
-1.1102E-14 -1.0374E-12 -7.9936E-14 4.6629E-14 -5.7621E-14
3.3751E-14 -6.9278E-14 1.0000E+00 1.4211E-13 1.9540E-14
5.5422E-13 -3.9080E-14 -1.1058E-13 4.0412E-14 -1.1990E-14
4.6185E-14 -5.7980E-12 6.2528E-13 -8.1712E-14 7.9936E-15
-2.4514E-13 1.8474E-13 5.9686E-13 1.0000E+00 -8.5265E-14
-5.1159E-12 -5.6843E-13 2.3981E-14 7.8160E-14 4.9738E-14
-7.3896E-13 -2.0009E-11 -4.7748E-12 1.3642E-12 -6.1462E-13
1.3642E-12 -3.4106E-12 1.5461E-11 -5.6843E-14 1.0000E+00
MATRIX DC* HAS 15 ROWS AND 1 COLUMNS
-1.9149E-06
-1.8729E-06
-1.7436E-05
9.9519E-08
-7.6805E-06
-1.2673E-04
4.9877E-07
2.9333E-06
4.5277E-06
6.6766E-07
2.6512E-06
1.8164E-05
-3.6874E-07
1.8826E-06
-1.1702E-05

```

APPENDIX B – Concluded

Parameter	Group	Row	Column A priori	Deviation	Scale	Answer	Deviation
1	1	1	1	-0.000099999.0000	1.0000	-.2740	.0975
-0	5	1	1	-0.0000	-0.0000	1.0000	-0.0000
1	1	1	2	-0.000099999.0000	1.0000	.0630	.0139
-0	5	1	2	-0.0000	-0.0000	1.0000	-0.0000
1	1	1	3	-0.000099999.0000	1.0000	-11.4004	.0065
-0	5	1	3	-0.0000	-0.0000	1.0000	-0.0000
1	2	1	1	-0.000099999.0000	1.0000	8.2232	.0259
-0	6	1	1	-0.0000	-0.0000	1.0000	-0.0000
1	2	1	2	-0.000099999.0000	1.0000	4.1772	.0118
-0	6	1	2	-0.0000	-0.0000	1.0000	-0.0000
1	2	1	3	-0.000099999.0000	1.0000	.0019	.0037
-0	6	1	3	-0.0000	-0.0000	1.0000	-0.0000
1	1	2	1	-0.000099999.0000	1.0000	-.0013	.3813
-0	5	2	1	-0.0000	-0.0000	1.0000	-0.0000
1	1	2	2	-0.000099999.0000	1.0000	-.2081	.0678
-0	5	2	2	-0.0000	-0.0000	1.0000	-0.0000
1	1	2	3	-0.000099999.0000	1.0000	2.1701	.0355
-0	5	2	3	-0.0000	-0.0000	1.0000	-0.0000
1	2	2	1	-0.000099999.0000	1.0000	-.1406	.0631
-0	6	2	1	-0.0000	-0.0000	1.0000	-0.0000
1	2	2	2	-0.000099999.0000	1.0000	-3.7614	.0251
-0	6	2	2	-0.0000	-0.0000	1.0000	-0.0000
1	2	2	3	-0.000099599.0000	1.0000	-.0009	.0240
-0	6	2	3	-0.0000	-0.0000	1.0000	-0.0000
1	1	3	1	-0.000099999.0000	1.0000	.0830	.3457
-0	5	3	1	-0.0000	-0.0000	1.0000	-0.0000
1	1	3	2	-0.000099999.0000	1.0000	-.3149	.0516
-0	5	3	2	-0.0000	-0.0000	1.0000	-0.0000
1	2	3	3	-0.000099999.0000	1.0000	.0004	.0221
-0	6	3	3	-0.0000	-0.0000	1.0000	-0.0000

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13.01.09. LRC COMPUTER COMPLEX
13.01.09.JOB,1,300,77777,1000. A2329 R
13.01.09.KN211 1150
13.01.09.USER.TAYLOR, LAWRENCE W , JR 0008
13.01.09.75539N 14110
13.01.09.RUN(S)
13.01.16.LINECNT(10000)
13.01.16.SETINDF.
13.01.17.LGO.
13.01.25.MEMORY 035600 CM
13.01.25.RFL 0000085 O/S CALLS
13.01.25.RFL CPU 3.589856 SEC.
13.03.05.STOP
13.03.05.SPPRINT(OUTPUT,5)
13.03.06. 0000103 O/S CALLS
13.03.06.CPU 101.891553 SEC.
13.03.07.PPU 15.654912 SEC.
13.08.29. LB91674. 1813 LINES PRINTED. LP25

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